

**SC705: Advanced Statistics**  
**Instructor: Natasha Sarkisian**  
**Class notes: Model Building Strategies**

### **Model Diagnostics**

The model diagnostics and improvement strategies discussed here apply to both measurement and structural models. Remember that you should always examine your data and perform the necessary transformations before you start estimating SEM – explore univariate distributions, bivariate relationships, and multivariate models (using OLS techniques). Only after you fixed various potential problems should you proceed to estimating SEM. After you estimated such a model, there are some additional diagnostics to consider.

#### I. Examining individual estimates

##### 1. Assessing the parameter estimates

One of the first steps is to determine the viability of the estimated values of parameters – they should exhibit the correct sign and size, and be consistent with the underlying theory. Any estimates falling outside the acceptable range (e.g. correlations higher than 1, negative variances – known as Heywood cases) indicate a problem with the model.

##### 2. Assessing the standard errors

Another indicator of poor model fit is the presence of standard errors that are excessively large or small. If they approximate zero, the test statistic for the parameter cannot be defined; if they are extremely large, this means the parameter cannot be determined. There are no clear guidelines as to what's too large or too small because standard errors are influenced by the units of measurement of the respective variables. Inaccurate standard errors are especially common when analyses are based on the correlation matrix.

##### 3. Statistical significance of parameter estimates

Nonsignificant parameters, with the exception of error variances, can be considered unimportant to the model, and, in the interest of parsimony, they should be deleted (provided there is a sufficient sample size to be able to rely on significance testing).

##### 4. Squared multiple correlations

For the structural model, two sets of squared multiple correlations are calculated – a set calculated from the structural model and a set calculated from the reduced form model. Those from the structural model are the  $R^2$  values indicating the % of variance in each endogenous variable explained by all the variables used in its model (both exogenous and other endogenous – i.e. it takes into account both betas and gammas). Those from the reduced form model are the  $R^2$  values indicating the % of variance in each endogenous variable explained by the exogenous variables only (note: reduced form model recalculates the equations to express the endogenous variables solely in terms of the exogenous ones). It is more appropriate to report and interpret reduced form  $R^2$ , especially if you deal with non-recursive models or correlated disturbance terms.

For the measurement model (for Xs and for Ys), squared multiple correlations are  $R^2$  values indicating the % of variance of each indicator explained by the latent factor. They serve as reliability indicators of the extent to which each observed variable measures its latent factor.

5. Watch out for warning messages.

Not only you should watch for estimated parameter values that are not plausible, but also for warning messages that can be in the middle of the output, e.g.:

W\_A\_R\_N\_I\_N\_G: PHI is not positive definite

This message also indicates that there are some values that are not plausible, but in this case it is not a specific value that is not plausible that a combination of values -- values of variances and covariances in the PHI matrix are such that they could not be plausibly occurring at the same time. This usually happens when you forget to fix or free something that you should. For example, this can happen if you do not fix one indicator per factor to have a lambda of 1.

Also, when selecting which indicator to fix to 1, it largely doesn't matter which one -- the only thing that this affects is the units of measurement for the variance of the corresponding latent variable. If all the indicators are measured on the same scale, just pick any. But if they have different scales, you might want to pick one that is measured on a scale similar to the indicators that you selected to have a lambda=1 for the other latent variables.

Let's say you are measuring class with years of education and income. If you pick years of education, variance will be in years of education squared, but if you pick income and it is in dollars, variance will be in dollars squared and can result in huge numbers. If you have one variance much larger or much smaller than the rest of variances (for other latent variables), you can potentially run into a problem when estimating the model because the iteration process will have difficulties converging. Therefore, you should try to pick similar units. If you have no choice and have to select income, divide it by 1000 or by 10000 to make the units more proportional to the units of indicators of other latent variables.

## II. Assessing model as a whole -- goodness-of-fit statistics

A range of goodness-of-fit statistics exists for SEM (see handout, pp.240-241 from Maruyama, Geoffrey M. 1998. Basics of Structural Equation Modeling. Thousand Oaks, CA: Sage Publications). This diversity can be quite confusing. We'll discuss a few indices and the guidelines for using them.

1. Chi-square is the likelihood ratio test statistic. It tests the null hypothesis that the variance-covariance matrix estimated from our model doesn't differ from the observed one (i.e. that all residuals are zero). This test, however, is sensitive to sample size and detects even minor deviations when the sample size is large). Some also use chi-square to d.f. ratio -- a smaller ratio indicates a better fit.

2. Non-centrality parameter (NCP, symbolized by  $\lambda$ ) is a measure of discrepancy between the observed variance-covariance matrix and the estimated one. It is therefore a measure of “badness-of-fit.”

3. RMSEA (Root Mean Square Error of Approximation). This goodness-of-fit index asks “How well would the model fit the population covariance matrix if it were available?” It measures that discrepancy per degree of freedom. Values less than .05 indicate good fit, and values as high as .08 represent reasonable errors of approximation in the population. .08-.10 indicates mediocre fit, and greater than .10 – poor fit. LISREL also reports the confidence interval for RMSEA that should be taken into account when making a judgment. LISREL also provides a p-value for this statistic; the suggested cutoff for p-value, however, is  $>.50$ . This index can be used to compare non-nested models (nested models are those that have similar structure, with the only difference being the number of free parameters).

4. Expected Cross-Validation Index (ECVI). This index tries to assess, in a single sample, the likelihood that the model cross-validates across similar-sized samples from the same population. It is usually used in a multiple-model setup, where the model with the smallest ECVI has the greatest potential for replication. At the very least, we can compare it with the values for a saturated model (the least restricted, just-identified model) and independence model (the most restricted model -- model assuming null correlations among all variables in the model). This index can also be used to compare non-nested models.

5. AIC and CAIC (Akaike’s Information Criterion and Consistent Akaike’s Information Criterion). These criteria address the issue of parsimony, combining the goodness-of-fit measure with the information on the number of estimated parameters. AIC carries a penalty related to the degrees of freedom but not the sample size, while CAIC takes the sample size into account as well. These two indices are usually used when comparing multiple models. The smaller values represent better fit. These indices can be used to compare non-nested models.

6. NFI (Normed Fit Index) has been the criterion of choice for a long time, but recent evidence showed that it has tendency to underestimate fit in small samples. This index compares fits of two different (nested) models (the default presented in LISREL is the null model). Values for NFI range from zero to 1. A values of  $>.90$  indicates an acceptable fit. The NNFI takes the complexity of model (number of parameters) into account, but because it’s not normed (can go beyond 1) it is difficult to interpret. The Parsimony Normed Fit Index (PNFI) also attempts to adjust the NFI; it is normed but typically, parsimony-based indices have substantially lower values that the threshold levels generally perceived as acceptable for other normed indices of fit. The CFI (Comparative Fit Index) takes sample size into account so it avoids the problems of NFI. The numeric value of CFI is interpreted the same way as for NFI. The IFI (Incremental Index of Fit) was developed to address both the issue of parsimony and sample size, it is also interpreted the same way. Finally, the Relative Fit Index (RFI) is algebraically equivalent to CFI.

7. Critical N (CN) focuses directly on the adequacy of sample size rather than on model fit. It tests what sample size would be sufficient to yield an adequate model fit for a chi-square test. CN value in excess of 200 is indicative of a model that adequately represents the sample data.

8. The Root Mean Square Residual (RMR) represents the average residual value. It is best interpreted in the metric of correlation matrix (i.e. it is the residual for correlations rather than covariances). In a well-fitting model, this value will be small – .05 or less.

9. GFI (Goodness of Fit Index) is an absolute measure – it measures the amount of variance and covariance explained by the model (compared with null model). AGFI is similar, but it adjusts for the number of degrees of freedom in the specified model (i.e. accounts for parsimony). Both indices range from 0 to 1, with values close to 1 being indicative of good fit (although theoretically, it is possible for them to be negative as well when the model is worse than no model at all). PGFI (Parsimony Goodness of Fit Index) takes into account the number of estimated parameters when assessing goodness-of-fit. As mentioned above, parsimony-based indices have substantially lower values than the threshold levels generally perceived as acceptable for other normed indices of fit.

### III. Evaluating model misspecifications

To identify potential model misfit, one can examine residuals and modification indices. So far, we have not been obtaining them in the output, but in fact, we can use a number of useful output options. These are specified in the OU command. Some of them regulate what is included in the output:

RS - Outputs residuals, standardized residuals, QQ-plot, and fitted covariance matrix.

EF - Outputs total and indirect effects.

MI - Outputs modification indices.

SS - Outputs the standardized solution.

SC - Outputs the completely standardized solution.

ALL - Outputs all the results.

ND=<number> -- Specifies the number of decimals for the results; default ND=2

Other options allow you to print the results into separate files:

<option> = <filename>

Options correspond to the names of matrices, e.g. LX, LY, BE, GA, PH, PS, etc. The folder name of the text file may be omitted if the text file and the LISREL syntax file are in the same folder.

NP=<number> -- Specifies the number of decimals for external text files to be produced; default: NP=3

OU command also includes some estimation options, e.g.:

IT=<number> -- Specifies the maximum number of iterations for the iterative algorithm; default: IT=five times the number of free parameters.

AD=<number> -- Specifies the iteration number at which the admissibility of the solution will be checked and the iterations will stop if the check fails; default: AD=20. This check may be turned off with the specification AD=OFF.

AM - Invokes the automatic model modification procedure. If this option is present, the program will modify the model sequentially by freeing at each step the fixed or constrained parameter that

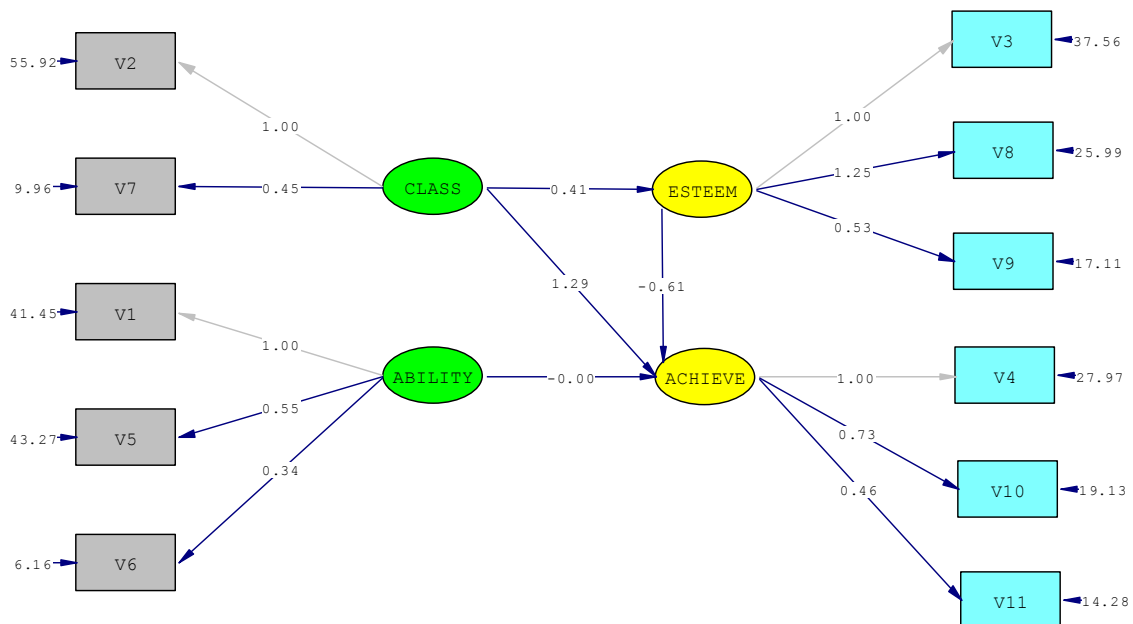
has the largest modification index. You should be really careful with this option, however, because the resulting model will be atheoretical.

Let's look at residuals and modification indices for our last example—add RS and MI to OU.

```

DA NI=11 NO=414 MA=CM
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.345 1.00
.287 .377 1.00
.252 .579 .114 1.00
.637 .335 .253 .255 1.00
.768 .339 .302 .278 .600 1.00
.254 .703 .337 .591 .312 .313 1.00
.166 .429 .724 .143 .264 .216 .364 1.00
.104 .411 .506 .142 .199 .163 .551 .630 1.00
.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00
.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00
SD
13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9
SE
3 8 9 4 10 11 2 7 1 5 6
MO NX=5 NK=2 NY=6 NE=2 LX=FU, FI LY=FU, FI PH=SY,FR PS=DI,FR TD=DI,FR TE=DI,FR
BE=FU,FI GA=FU,FR
LK
CLASS ABILITY
LE
ESTEEM ACHIEVE
FR LX 2 1 LX 4 2 LX 5 2 LY 2 1 LY 3 1 LY 5 2 LY 6 2 BE 2 1
FI GA 1 2
VA 1.0 LX 1 1 LX 3 2 LY 1 1 LY 4 2
PD
OU RS MI

```



### 1. Residuals

Fitted Covariance Matrix

	V3	V8	V9	V4	V10	V11
V3	100.00					
V8	77.91	123.21				
V9	33.24	41.48	34.81			
V4	27.94	34.87	14.88	252.80		
V10	20.42	25.48	10.87	164.33	139.24	
V11	12.93	16.13	6.88	104.03	76.03	62.41
V2	51.13	63.80	27.22	128.58	93.98	59.49
V7	22.92	28.60	12.20	57.65	42.13	26.67
V1	25.21	31.46	13.42	63.19	46.19	29.24
V5	13.74	17.15	7.32	34.44	25.17	15.94
V6	8.65	10.79	4.60	21.67	15.84	10.03

Fitted Covariance Matrix

	V2	V7	V1	V5	V6
V2	179.56				
V7	55.43	34.81			
V1	60.97	27.33	193.21		
V5	33.23	14.90	82.72	88.36	
V6	20.91	9.37	52.04	28.37	24.01

Fitted Residuals

	V3	V8	V9	V4	V10	V11
V3	0.00					
V8	2.45	0.00				
V9	-3.39	-0.22	0.00			
V4	-9.82	-9.63	-1.55	0.01		
V10	3.41	12.50	2.98	0.78	0.00	
V11	-2.90	-2.54	4.02	-0.15	-0.71	0.00
V2	-0.61	0.01	5.27	-5.22	1.05	-3.81
V7	-3.04	-4.77	6.98	-2.20	1.10	4.84
V1	14.68	-5.85	-4.89	-7.50	-5.67	-6.40
V5	10.04	10.40	3.72	3.67	6.77	2.85
V6	6.15	0.96	0.11	-0.01	2.03	-1.08

Fitted Residuals

	V2	V7	V1	V5	V6
V2	0.00				
V7	0.15	0.00			
V1	3.29	-6.50	0.00		
V5	8.96	2.40	0.51	0.00	
V6	1.35	-0.32	0.27	-0.73	0.00

Summary Statistics for Fitted Residuals

Smallest Fitted Residual = -9.82  
 Median Fitted Residual = 0.00  
 Largest Fitted Residual = 14.68

Stemleaf Plot

```

- 8|86
- 6|554
- 4|87298
- 2|840952
- 0|6177632100000000000000
  0|113580114
  
```

2|045903477  
 4|083  
 6|280  
 8|0  
 10|04  
 12|5  
 14|7

Standardized Residuals

	V3	V8	V9	V4	V10	V11
V3	-	-				
V8	6.81	-				
V9	-5.15	-0.64	-			
V4	-2.30	-3.04	-0.52	1.10		
V10	1.04	4.89	1.32	3.02	1.10	
V11	-1.20	-1.20	2.50	-0.43	-2.12	1.10
V2	-0.18	0.00	2.30	-1.93	0.48	-2.01
V7	-2.15	-4.12	7.08	-1.97	1.20	5.99
V1	2.65	-1.01	-1.44	-1.68	-1.61	-2.29
V5	2.49	2.41	1.53	0.77	1.88	1.12
V6	3.12	0.46	0.09	-0.01	1.50	-1.04

Standardized Residuals

	V2	V7	V1	V5	V6
V2	-	-			
V7	1.09	-			
V1	0.79	-3.77	-		
V5	2.20	1.37	0.85	-	
V6	0.86	-0.49	2.59	-2.73	-

Summary Statistics for Standardized Residuals

Smallest Standardized Residual = -5.15  
 Median Standardized Residual = 0.00  
 Largest Standardized Residual = 7.08

Stemleaf Plot

- 4|11  
 - 2|807332100  
 - 0|97642200655420000000000  
 0|1558889011111234559  
 2|234556701  
 4|9  
 6|081

Largest Negative Standardized Residuals

Residual for V9 and V3 -5.15  
 Residual for V4 and V8 -3.04  
 Residual for V7 and V8 -4.12  
 Residual for V1 and V7 -3.77  
 Residual for V6 and V5 -2.73

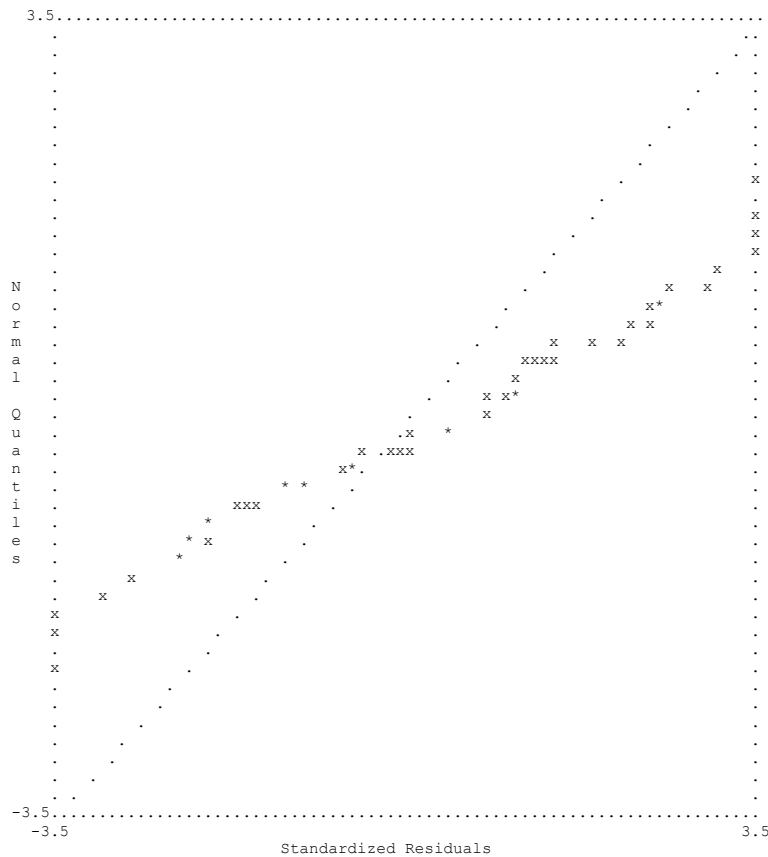
Largest Positive Standardized Residuals

Residual for V8 and V3 6.81  
 Residual for V10 and V8 4.89  
 Residual for V10 and V4 3.02  
 Residual for V7 and V9 7.08  
 Residual for V7 and V11 5.99  
 Residual for V1 and V3 2.65  
 Residual for V6 and V3 3.12

Residual for V6 and V1 2.59

DA NI=11 NO=414 MA=CM

Qplot of Standardized Residuals



Using this information, we evaluate the normality of residuals distribution, and assess whether the distribution is symmetric. The Q-plot is especially useful: deviations from the straight line may indicate that (a) the model is in some ways misspecified, (b) the data are non-normally distributed, or (c) there are some nonlinear relationships. Note that along with regular residuals, we obtained standardized residuals. These are residuals divided by their standard error. These are, therefore, analogous to z-scores. Values  $>2.58$  are considered large. These are indicative of possible fit problems in the model.

2. Modification indices. These can be conceptualized as individual chi-square statistics for specific parameters (with 1 d.f., so the critical value is 3.841 for .05 level, 6.635 for .01 level, 10.828 for .001 level). They are estimated for all the parameters that were not freely estimated (i.e. were fixed), and represent the expected drop in the overall chi-square if the parameter were to be freely estimated. Therefore, all freely estimated parameters automatically have MI values equal to zero. Let's examine these for our last model:

DA NI=11 NO=414 MA=CM

Modification Indices and Expected Change



Modification Indices for LAMBDA-Y

	ESTEEM	ACHIEVE
	-----	-----
V3	- -	1.11
V8	- -	0.00
V9	- -	1.51
V4	23.13	- -
V10	26.98	- -
V11	0.06	- -

Expected Change for LAMBDA-Y

	ESTEEM	ACHIEVE
	-----	-----
V3	- -	-0.03
V8	- -	0.00
V9	- -	0.02
V4	-0.24	- -
V10	0.20	- -
V11	-0.01	- -

Modification Indices for LAMBDA-X

	CLASS	ABILITY
	-----	-----
V2	- -	2.83
V7	- -	5.16
V1	7.47	- -
V5	6.71	- -
V6	0.72	- -

Expected Change for LAMBDA-X

	CLASS	ABILITY
	-----	-----
V2	- -	0.08
V7	- -	-0.05
V1	-0.14	- -
V5	0.10	- -
V6	0.02	- -

Modification Indices for BETA

	ESTEEM	ACHIEVE
	-----	-----
ESTEEM	- -	1.19
ACHIEVE	- -	- -

Expected Change for BETA

	ESTEEM	ACHIEVE
	-----	-----
ESTEEM	- -	-23.02
ACHIEVE	- -	- -

Modification Indices for GAMMA

	CLASS	ABILITY
	-----	-----
ESTEEM	- -	1.19
ACHIEVE	- -	- -

	Expected Change for GAMMA	
	CLASS	ABILITY
	-----	-----
ESTEEM	- -	0.04
ACHIEVE	- -	- -

No Non-Zero Modification Indices for PHI

No Non-Zero Modification Indices for PSI

Modification Indices for THETA-EPS						
	V3	V8	V9	V4	V10	V11
	-----	-----	-----	-----	-----	-----
V3	- -					
V8	46.35	- -				
V9	26.49	0.40	- -			
V4	0.24	4.20	2.76	- -		
V10	0.02	48.72	16.05	8.99	- -	
V11	0.85	12.42	17.01	0.19	4.52	- -

Expected Change for THETA-EPS						
	V3	V8	V9	V4	V10	V11
	-----	-----	-----	-----	-----	-----
V3	- -					
V8	44.70	- -				
V9	-12.17	-1.98	- -			
V4	1.15	-4.92	-2.50	- -		
V10	-0.28	12.88	-4.64	15.06	- -	
V11	-1.30	-5.07	3.74	-1.20	-4.20	- -

Modification Indices for THETA-DELTA-EPS						
	V3	V8	V9	V4	V10	V11
	-----	-----	-----	-----	-----	-----
V2	0.01	1.30	1.81	3.76	0.25	15.18
V7	3.74	32.76	88.24	9.62	6.31	59.78
V1	10.56	4.66	6.10	3.32	5.55	0.21
V5	4.12	6.61	0.75	2.07	0.94	1.47
V6	1.57	0.04	0.00	0.16	5.30	2.69

Expected Change for THETA-DELTA-EPS						
	V3	V8	V9	V4	V10	V11
	-----	-----	-----	-----	-----	-----
V2	0.33	3.74	-2.61	6.03	1.18	-7.04
V7	-2.58	-8.18	7.88	-4.19	-2.58	6.04
V1	9.29	-6.30	-4.54	5.21	-5.19	-0.78
V5	-4.82	6.23	1.33	-3.41	1.77	1.74
V6	1.29	-0.21	-0.03	-0.41	1.82	-1.02

Modification Indices for THETA-DELTA					
	V2	V7	V1	V5	V6
	-----	-----	-----	-----	-----
V2	- -				
V7	1.19	- -			
V1	6.72	7.05	- -		
V5	0.08	0.60	0.72	- -	
V6	2.09	0.68	6.71	7.45	- -

Expected Change for THETA-DELTA					
	V2	V7	V1	V5	V6
	-----	-----	-----	-----	-----
V2	- -				
V7	10.75	- -			
V1	9.54	-4.23	- -		

V5	-0.83	1.01	6.08	-	-	
V6	-1.91	0.47	15.61	-6.58		- -

Maximum Modification Index is 88.24 for Element ( 2, 3) of THETA DELTA-EPSILON

Time used: 0.047 Seconds

The modification indices reflect the predicted changes in chi-square, and the expected changes reflect the predicted changes in the coefficients. Based on these, we can respecify the model, but we should be careful because there is always a risk of overfitting the model to the data.

### Interpreting the Results: Indirect and Total Effects and Standardized Coefficients

Three output options that are useful for interpreting the results are EF, SS and SC:

```

DA NI=11 NO=414 MA=CM
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.345 1.00
.287 .377 1.00
.252 .579 .114 1.00
.637 .335 .253 .255 1.00
.768 .339 .302 .278 .600 1.00
.254 .703 .337 .591 .312 .313 1.00
.166 .429 .724 .143 .264 .216 .364 1.00
.104 .411 .506 .142 .199 .163 .551 .630 1.00
.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00
.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00
SD
13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9
SE
3 8 9 4 10 11 2 7 1 5 6
MO NX=5 NK=2 NY=6 NE=2 LX=FU, FI LY=FU, FI PH=SY,FR PS=DI,FR TD=DI,FR TE=DI,FR
BE=FU,FI GA=FU,FR
LK
CLASS ABILITY
LE
ESTEEM ACHIEVE
FR LX 2 1 LX 4 2 LX 5 2 LY 2 1 LY 3 1 LY 5 2 LY 6 2 BE 2 1
FI GA 1 2
VA 1.0 LX 1 1 LX 3 2 LY 1 1 LY 4 2
PD
OU EF SC SS

```

[Other output omitted]

Standardized Solution

	LAMBDA-Y	
	ESTEEM	ACHIEVE
	-----	-----
V3	7.90	- -
V8	9.86	- -
V9	4.21	- -
V4	- -	14.99
V10	- -	10.96
V11	- -	6.94

LAMBDA-X		
	CLASS	ABILITY
	-----	-----
V2	11.12	- -
V7	4.99	- -
V1	- -	12.32
V5	- -	6.71
V6	- -	4.22

BETA

	ESTEEM	ACHIEVE
	-----	-----
ESTEEM	- -	- -
ACHIEVE	-0.32	- -

GAMMA

	CLASS	ABILITY
	-----	-----
ESTEEM	0.58	- -
ACHIEVE	0.96	0.00

Correlation Matrix of ETA and KSI

	ESTEEM	ACHIEVE	CLASS	ABILITY
	-----	-----	-----	-----
ESTEEM	1.00			
ACHIEVE	0.24	1.00		
CLASS	0.58	0.77	1.00	
ABILITY	0.26	0.34	0.45	1.00

PSI

Note: This matrix is diagonal.

	ESTEEM	ACHIEVE
	-----	-----
	0.66	0.34

Regression Matrix ETA on KSI (Standardized)

	CLASS	ABILITY
	-----	-----
ESTEEM	0.58	- -
ACHIEVE	0.77	0.00

DA NI=11 NO=414 MA=CM

Completely Standardized Solution

LAMBDA-Y

	ESTEEM	ACHIEVE
	-----	-----
V3	0.79	- -
V8	0.89	- -
V9	0.71	- -
V4	- -	0.94
V10	- -	0.93
V11	- -	0.88

LAMBDA-X

	CLASS	ABILITY
--	-------	---------

V2	0.83	- -
V7	0.84	- -
V1	- -	0.89
V5	- -	0.71
V6	- -	0.86

BETA

	ESTEEM	ACHIEVE
	-----	-----
ESTEEM	- -	- -
ACHIEVE	-0.32	- -

GAMMA

	CLASS	ABILITY
	-----	-----
ESTEEM	0.58	- -
ACHIEVE	0.96	0.00

Correlation Matrix of ETA and KSI

	ESTEEM	ACHIEVE	CLASS	ABILITY
	-----	-----	-----	-----
ESTEEM	1.00			
ACHIEVE	0.24	1.00		
CLASS	0.58	0.77	1.00	
ABILITY	0.26	0.34	0.45	1.00

PSI

Note: This matrix is diagonal.

	ESTEEM	ACHIEVE
	-----	-----
	0.66	0.34

THETA-EPS

	V3	V8	V9	V4	V10	V11
	-----	-----	-----	-----	-----	-----
	0.38	0.21	0.49	0.11	0.14	0.23

THETA-DELTA

	V2	V7	V1	V5	V6
	-----	-----	-----	-----	-----
	0.31	0.29	0.21	0.49	0.26

Regression Matrix ETA on KSI (Standardized)

	CLASS	ABILITY
	-----	-----
ESTEEM	0.58	- -
ACHIEVE	0.77	0.00

DA NI=11 NO=414 MA=CM

Total and Indirect Effects

Total Effects of KSI on ETA

	CLASS	ABILITY
	-----	-----
ESTEEM	0.41	- -
	(0.04)	
	9.83	
ACHIEVE	1.04	0.00
	(0.08)	(0.06)
	13.72	-0.03

Indirect Effects of KSI on ETA

	CLASS	ABILITY
	-----	-----
ESTEEM	- -	- -
ACHIEVE	-0.25	- -
	(0.06)	
	-4.52	

Total Effects of ETA on ETA

	ESTEEM	ACHIEVE
	-----	-----
ESTEEM	- -	- -
ACHIEVE	-0.61	- -
	(0.11)	
	-5.52	

Largest Eigenvalue of B\*B' (Stability Index) is 0.373

Total Effects of ETA on Y

	ESTEEM	ACHIEVE
	-----	-----
V3	1.00	- -
V8	1.25	- -
	(0.07)	
	16.97	
V9	0.53	- -
	(0.04)	
	14.60	
V4	-0.61	1.00
	(0.11)	
	-5.52	
V10	-0.45	0.73
	(0.08)	(0.02)
	-5.52	34.47
V11	-0.28	0.46
	(0.05)	(0.02)
	-5.49	29.44

Indirect Effects of ETA on Y

	ESTEEM	ACHIEVE
	-----	-----
V3	- -	- -
V8	- -	- -
V9	- -	- -
V4	-0.61	- -
	(0.11)	
	-5.52	
V10	-0.45	- -
	(0.08)	
	-5.52	
V11	-0.28	- -

(0.05)  
-5.49

Total Effects of KSI on Y

	CLASS	ABILITY
	-----	-----
V3	0.41 (0.04) 9.83	- -
V8	0.52 (0.05) 10.45	- -
V9	0.22 (0.02) 9.42	- -
V4	1.04 (0.08) 13.72	0.00 (0.06) -0.03
V10	0.76 (0.06) 13.60	0.00 (0.04) -0.03
V11	0.48 (0.04) 13.19	0.00 (0.03) -0.03

DA NI=11 NO=414 MA=CM

Standardized Total and Indirect Effects

Standardized Total Effects of KSI on ETA

	CLASS	ABILITY
	-----	-----
ESTEEM	0.58	- -
ACHIEVE	0.77	0.00

Standardized Indirect Effects of KSI on ETA

	CLASS	ABILITY
	-----	-----
ESTEEM	- -	- -
ACHIEVE	-0.19	- -

Standardized Total Effects of ETA on ETA

	ESTEEM	ACHIEVE
	-----	-----
ESTEEM	- -	- -
ACHIEVE	-0.32	- -

Standardized Total Effects of ETA on Y

	ESTEEM	ACHIEVE
	-----	-----
V3	7.90	- -
V8	9.86	- -
V9	4.21	- -
V4	-4.83	14.99
V10	-3.53	10.96
V11	-2.23	6.94

Completely Standardized Total Effects of ETA on Y

	ESTEEM	ACHIEVE
	-----	-----
V3	0.79	- -
V8	0.89	- -
V9	0.71	- -
V4	-0.30	0.94
V10	-0.30	0.93
V11	-0.28	0.88

Standardized Indirect Effects of ETA on Y

	ESTEEM	ACHIEVE
	-----	-----
V3	- -	- -
V8	- -	- -
V9	- -	- -
V4	-4.83	- -
V10	-3.53	- -
V11	-2.23	- -

Completely Standardized Indirect Effects of ETA on Y

	ESTEEM	ACHIEVE
	-----	-----
V3	- -	- -
V8	- -	- -
V9	- -	- -
V4	-0.30	- -
V10	-0.30	- -
V11	-0.28	- -

Standardized Total Effects of KSI on Y

	CLASS	ABILITY
	-----	-----
V3	4.60	- -
V8	5.74	- -
V9	2.45	- -
V4	11.57	-0.02
V10	8.46	-0.02
V11	5.35	-0.01

Completely Standardized Total Effects of KSI on Y

	CLASS	ABILITY
	-----	-----
V3	0.46	- -
V8	0.52	- -
V9	0.41	- -
V4	0.73	0.00
V10	0.72	0.00
V11	0.68	0.00

Time used: 0.016 Seconds

## Variables with single indicators

So far in our model all variables were latent variables with multiple indicators. But sometimes we have to use single-indicator variables in our analysis – e.g. if we have to rely on income only to measure class (or to include gender, or age into the model). An easy way to do that is to specify a single-indicator latent variable with the corresponding  $\lambda$  fixed to 1, and the



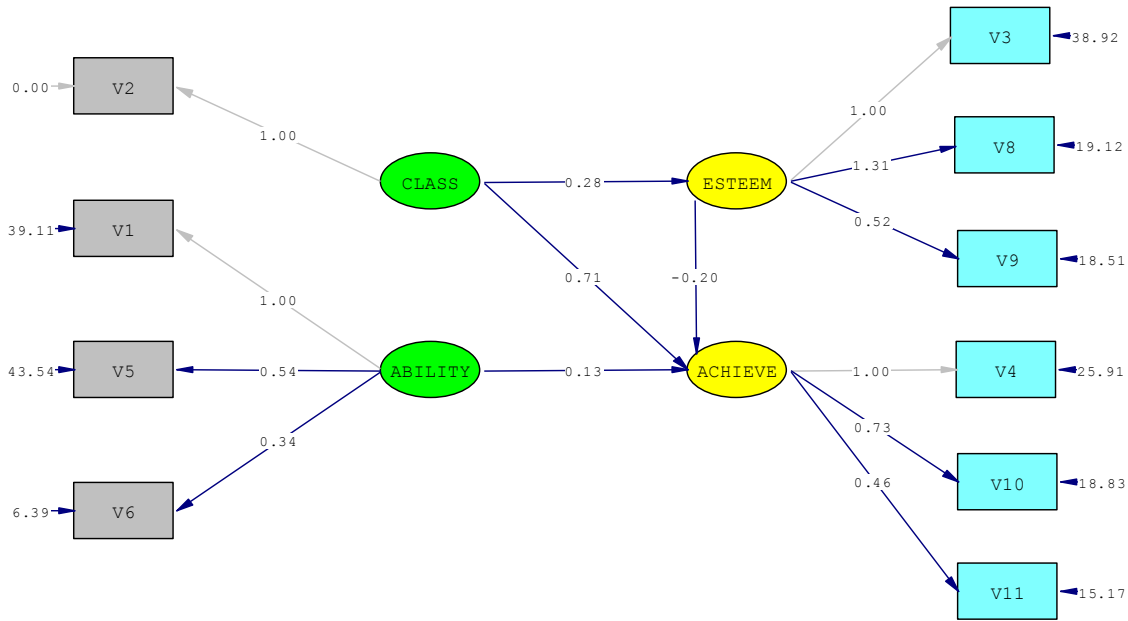
corresponding measurement error  $\delta$  fixed to 0 (i.e., we assume no measurement error). E.g. if only V2 is available as a measure of class, we specify:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & \lambda_{32} \\ 0 & \lambda_{42} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix}$$

```

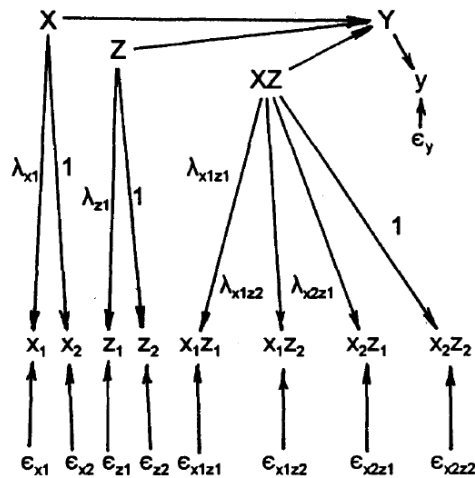
DA NI=11 NO=414 MA=CM
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.345 1.00
.287 .377 1.00
.252 .579 .114 1.00
.637 .335 .253 .255 1.00
.768 .339 .302 .278 .600 1.00
.254 .703 .337 .591 .312 .313 1.00
.166 .429 .724 .143 .264 .216 .364 1.00
.104 .411 .506 .142 .199 .163 .551 .630 1.00
.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00
.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00
SD
13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9
SE
3 8 9 4 10 11 2 1 5 6 /
MO NX=4 NK=2 NY=6 NE=2 LX=FU,FI LY=FU,FI PH=SY,FR PS=DI,FR TD=DI,FR TE=DI,FR
BE=FU,FI GA=FU,FR
LK
CLASS ABILITY
LE
ESTEEM ACHIEVE
FR LX 3 2 LX 4 2 LY 2 1 LY 3 1 LY 5 2 LY 6 2 BE 2 1
FI GA 1 2 TD 1 1
VA 1.0 LX 1 1 LX 2 2 LY 1 1 LY 4 2
PD
OU

```



### Variables with Interactions or Quadratic Terms

Interactions and squared terms create complications in SEM when they involve latent variables measured by multiple indicators because the same indicators are involved in measuring the interaction (or quadratic term) that are also involved in measuring the main effects, so lambdas and error terms are related and we need to specify that relationship. Typically, when specifying interactions for latent variables, we need to create interactions of each pair of indicators of variables X and Z involved in that interaction. If each of them has 2 indicators, that means 4 additional interaction indicators; if each has 3, that means 9 additional observed variables entered in the model. Those should be included in your variance-covariance matrix, and you need to link them to a latent variable XZ, as shown on the diagram:



(diagram from Robert A. Ping, 1996, "Latent Variable Interaction and Quadratic Effect Estimation," Psychological Bulletin 119: 166-175)

But still, we need to constrain the estimation so that lambdas are linked (as we can see, lambdas for XZ indicators are products of lambdas from the measurement of X and Z, and the measurement errors are also linked, but in a more complex way).

The easiest solution is the one proposed by Robert A. Ping (1996, "Latent Variable Interaction and Quadratic Effect Estimation," *Psychological Bulletin* 119: 166-175); it involves estimating the measurement model without interactions and squared terms and then calculating lambda coefficients and measurement error variances for the interaction terms based on the results of the initial measurement models and entering those numbers as constants (actual values) into the model with interactions and squared terms. As we know, we can use VA command in SEM to assign values for pretty much any parameter; what this means is we calculate values for lambdas for all XZ indicators as well as for measurement error variances for them and put in each of those numbers using VA command (separate VA statement for each of these parameters).

As is clear from the figure, the lambdas for each of products of indicators are calculated by multiplying lambdas from the measurement model. For example:

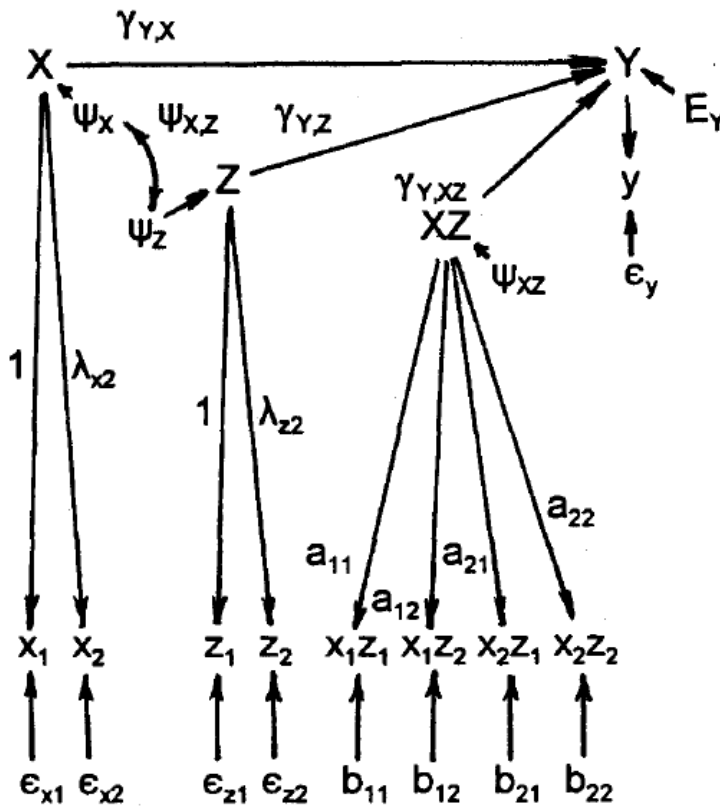


Figure 3. An interaction model using the proposed approach.  $a_{11} = \lambda_{x1}\lambda_{z1} = 1$ ,  $a_{12} = \lambda_{x1}\lambda_{z2} = \lambda_{z2}$ ;  $a_{21} = \lambda_{x2}\lambda_{z1} = \lambda_{x2}$ ,  $a_{22} = \lambda_{x2}\lambda_{z2}$ ; (diagram from Robert A. Ping, 1996, "Latent Variable Interaction and Quadratic Effect Estimation," *Psychological Bulletin* 119: 166-175)

But we also need to calculate variances of the error terms – variances of  $b_{11}$ ,  $b_{12}$ ,  $b_{21}$ , and  $b_{22}$ . We do that using the following formula:

$$\text{Var}(b) = K\lambda_x^2 \text{Var}(X) \text{Var}(\epsilon_z) + K\lambda_z^2 \text{Var}(Z) \text{Var}(\epsilon_x) + K \text{Var}(\epsilon_x) \text{Var}(\epsilon_z) + (2 - K)\lambda_x \lambda_z \text{Cov}(X, Z) \text{Cov}(\epsilon_x, \epsilon_z),$$

(K = 2 if x = z, K = 1 otherwise).

So K=2 if it's a quadratic term, and K=1 if it's an interaction. The last term,

$$(2 - K)\lambda_x \lambda_z \text{Cov}(X, Z) \text{Cov}(\epsilon_x, \epsilon_z),$$

is only used if the error terms of X indicators and Z indicators are allowed to be correlated. If they are not (as in the figure above), the formula is simplified to:

$$\text{Var}(b) = K\lambda_x^2 \text{Var}(X) \text{Var}(\epsilon_z) + K\lambda_z^2 \text{Var}(Z) \text{Var}(\epsilon_x) + K \text{Var}(\epsilon_x) \text{Var}(\epsilon_z)$$

So for our example above:

$$b_{11} = 1 * \lambda_{x1}^2 * \text{Var}(X) * \text{Var}(\epsilon_{z1}) + 1 * \lambda_{z1}^2 * \text{Var}(Z) * \text{Var}(\epsilon_{x1}) + 1 * \text{Var}(\epsilon_{x1}) * \text{Var}(\epsilon_{z1})$$

$$b_{12} = 1 * \lambda_{x1}^2 * \text{Var}(X) * \text{Var}(\epsilon_{z2}) + 1 * \lambda_{z2}^2 * \text{Var}(Z) * \text{Var}(\epsilon_{x1}) + 1 * \text{Var}(\epsilon_{x1}) * \text{Var}(\epsilon_{z2})$$

$$b_{21} = 1 * \lambda_{x2}^2 * \text{Var}(X) * \text{Var}(\epsilon_{z1}) + 1 * \lambda_{z1}^2 * \text{Var}(Z) * \text{Var}(\epsilon_{x2}) + 1 * \text{Var}(\epsilon_{x2}) * \text{Var}(\epsilon_{z1})$$

$$b_{22} = 1 * \lambda_{x2}^2 * \text{Var}(X) * \text{Var}(\epsilon_{z2}) + 1 * \lambda_{z2}^2 * \text{Var}(Z) * \text{Var}(\epsilon_{x2}) + 1 * \text{Var}(\epsilon_{x2}) * \text{Var}(\epsilon_{z2})$$

All the necessary values for this calculation are obtained from the LISREL output for the preliminary model – in addition to lambdas, we need variances of latent variables X and Z as well as measurement error variances of corresponding indicators of X and Z (x<sub>1</sub>, x<sub>2</sub>, z<sub>1</sub>, z<sub>2</sub>) – these measurement error variances are in theta delta matrix (on the diagonal).

Similar process works for a model with a quadratic term; for example:

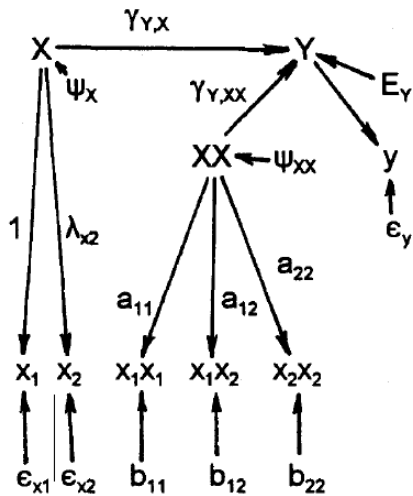


Figure 2. A quadratic model using the proposed approach.  $a_{11} = \lambda_{x1}\lambda_{x1} = 1$ ,  $a_{12} = \lambda_{x1}\lambda_{x2} = \lambda_{x2}$ ,  $a_{22} = \lambda_{x2}\lambda_{x2}$

(diagram from Robert A. Ping, 1996, "Latent Variable Interaction and Quadratic Effect Estimation," Psychological Bulletin 119: 166-175)

$$b_{11} = 4 * \lambda_{x1}^2 * \text{Var}(X) * \text{Var}(\epsilon_{x1}) + 2 * \text{Var}(\epsilon_{x1})^2$$

$$b_{12} = 2 * \lambda_{x1}^2 * \text{Var}(X) * \text{Var}(\epsilon_{x2}) + 2 * \lambda_{x2}^2 * \text{Var}(X) * \text{Var}(\epsilon_{x1}) + 2 * \text{Var}(\epsilon_{x1}) * \text{Var}(\epsilon_{x2})$$

$$b_{22} = 4 * \lambda_{x2}^2 * \text{Var}(X) * \text{Var}(\epsilon_{x2}) + 2 * \text{Var}(\epsilon_{x2})^2$$

Note that we should mean-center each of the indicators involved in an interaction or quadratic term before multiplying them by each other and use mean-centered indicators in our measurement model (if using covariance matrix, we should mean-center them before creating such a matrix). We don't standardize these indicators, just mean-center them -- that will preserve the variance but reduce collinearity.

Once we calculated all of these and have the actual numbers, we can create all the VA statements in our LISREL syntax, assign the numbers, and estimate our combined (measurement + structural) model that way. More specifically, if our interaction or squared term involves variables that are exogenous, we can just construct a traditional combined model with parameter values set as discussed above (all three variables -- X, Z and XZ will be exogenous). If the interaction or squared term involves at least one endogenous variable (or both), then, in order to allow for all three terms (X, Z, and XZ) or for the linear and quadratic term (X and XX) to be correlated, we need to consider all of them endogenous, even though some of them are not, and allow for their disturbance terms to be correlated in the PS matrix. [LISREL will allow you to define latent variables as endogenous even though nothing predicts them in the model.]

### **Obtaining Covariance Matrices and Using Raw Data**

Let's use nys2.sav data available in HLM folder to obtain the covariance matrix. First, we import nys2.sav into LISREL, creating nys2.PSF. Upon importing the data, we can obtain the covariance matrix – it will be in the file named nys.cov.

```
!Prelis syntax
SY='C:\nys2.PSF'
OU MA=CM SM=nys.cov
```

Note that here we obtained a covariance matrix using CM option; that's the standard when all variables are continuous. But if we are dealing with ordinal, categorical, or mixed data, we need two matrices, polychoric correlation matrix (PM) and asymptotic covariance matrix (AC), e.g.:

```
!Prelis syntax
SY='C:\nys2.PSF'
OU MA=PM SM=nys.pcm AC=asymptnys.acm
```

In the analysis syntax, in the DA statement, make sure to specify MA=PM and specify the following input matrices:

```
PM=nys.pcm
AC=asymptnys.acm
```

Further, our dataset contains missing values – we have to decide how to deal with those. Listwise deletion is used as default. We could, however, opt for using FIML method – Full Information Maximum Likelihood – which proceeds from raw data rather than from the covariance matrix. In that case, we make sure that missing values are identified in the .PSF file, and don't obtain any matrices in PRELIS. Instead, we input the raw data into LISREL by including the following into the DA statement:

```
RA=C:\nys2.PSF
```