Longitudinal Data Analysis
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Panel Data Analysis: Mixed Effects Models

So far, when analyzing panel data, we only allowed for the intercepts to vary across units (by having fixed effects or random effects for countries or individuals). A whole other class of models, mixed effects models, also known as multilevel models, hierarchical linear models, or growth curve models, allows for the coefficients themselves to vary across units. That is, we assume that the effects of time-varying variables, and time itself, are not the same across units. We will look at average effect of such variables, the extent to which there is variation around that average, and at level 2 (time-invariant) predictors that may explain that variation (so-called cross-level interactions). But first let's reexamine the equation for random effects model:
$Y_{ij} = \alpha + X\beta + u_i + e_{ij}$

We can also rewrite it as:
Level 1 model is: $Y_{ij} = \alpha + X\beta + e_{ij}$
Level 2 model is: $\alpha = \pi_0 + u_i$

Thus, we expressed a random effects model as a two-level model where we can explicitly see that the intercept for each unit equals to grand mean plus unit-specific residual. If our model also contains some time-invariant predictors, we can also write:
Level 1 model is: $Y_{ij} = \alpha + X\beta + e_{ij}$
Level 2 model is: $\alpha = \pi_0 + X_i\beta_i + u_i$

Moving beyond random effects models to mixed models, we can write a similar equation for each of level 1 regression coefficients:
Level 1 model is: $Y_{ij} = \alpha + X\beta + e_{ij}$
Level 2 model is: $\alpha = \pi_0 + X_i\beta_i + u_{0i}$,
$\beta_1 = \pi_1 + X_i\beta_i + u_{1i}$,
…

We will use an example that examines how attitudes toward deviant behavior change over time for teenagers, and what shapes that change. We will use a file called nys.dta. This file contains data for a cohort of adolescents in the National Youth Survey, ages 14 to 18. The dependent variable attit is a 9-item scale assessing attitudes favorable to deviant behavior (property damage, drug and alcohol use, stealing, etc.). The level-1 independent variables include: expo measuring exposure to deviant peers (students were asked how many of their friends engaged in the 9 deviant behaviors) and age (age in years). Level 2 include person-level variables: female, minority, and income.

```
. use http://www.sarkisian.net/socy7706/nys.dta
. reshape long attit expo, i(id) j(age)
(note: j = 14 15 16 17 18)
Data                               wide   ->   long
-----------------------------------------------------------------------------
Number of obs.                      241   ->    1205
Number of variables                  14   ->       7
```

```
j variable (5 values)                     ->   age
xij variables:
            attit14 attit15 ... attit18   ->   attit
               expo14 expo15 ... expo18   ->   expo
-----------------------------------------------------------------------------

. egen miss=rowmiss( attit expo)
. tab miss
       miss |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      1,066       88.46       88.46
          2 |        139       11.54      100.00
------------+-----------------------------------
      Total |      1,205      100.00

. drop if miss==2
(139 observations deleted)

. xtset id age, yearly
       panel variable:  id (unbalanced)
        time variable:  age, 14 to 18, but with gaps
                delta:  1 year
```

Remember: Data are considered strongly balanced if all the time points are the same and all cases are observed at all time points. Data are considered balanced if the cases have the same number of time values but these are not exactly the same time points. Data are unbalanced if cases are observed at different numbers of time points.

Focusing just on age, we could estimate a random effects model using both xtreg and mixed:

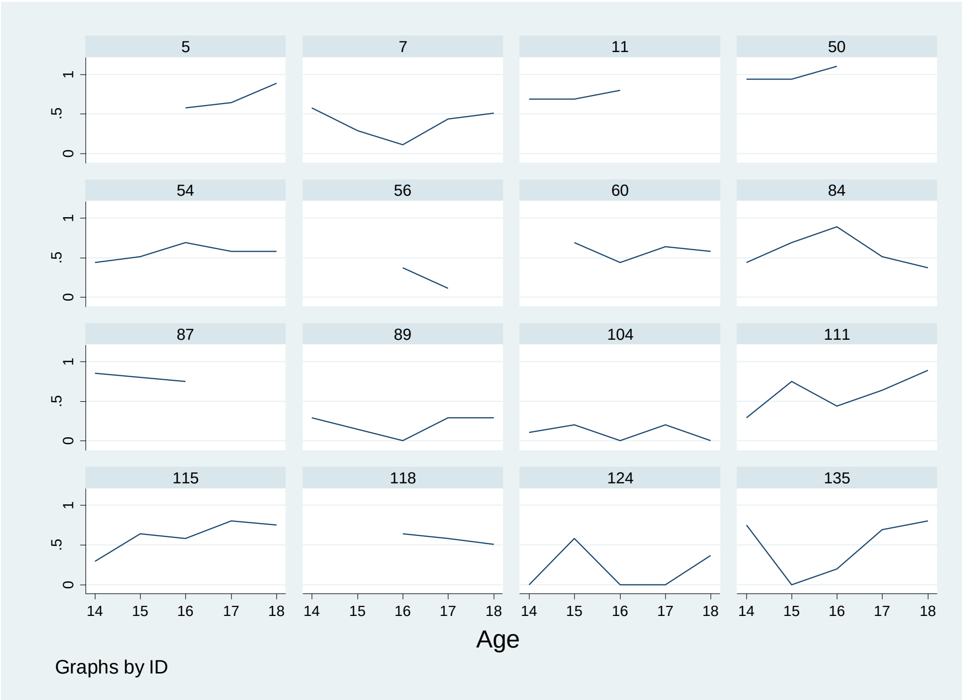
```
. xtreg attit age, re
Random-effects GLS regression                   Number of obs      =      1066
Group variable: id                              Number of groups   =       241

R-sq:  within  = 0.0674                         Obs per group: min =         1
       between = 0.0000                                        avg =       4.4
       overall = 0.0207                                        max =         5

Random effects u_i ~ Gaussian                   Wald chi2(1)       =     58.31
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0324074   .0042441     7.64   0.000     .0240892    .0407256
       _cons |  -.0258944   .0692441    -0.37   0.708    -.1616103    .1098215
-------------+----------------------------------------------------------------
     sigma_u |  .21445769
     sigma_e |  .18975623
         rho |   .5608825   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. mixed attit age || id:

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(1)      =      57.94
Log likelihood =  36.668959                     Prob > chi2       =     0.0000
```

```
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |    .032384   .0042543     7.61   0.000     .0240456    .0407223
       _cons |  -.0255082   .0693628    -0.37   0.713    -.1614569    .1104404
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                   var(_cons) |   .0443473   .0048941      .0357215    .0550558
-----------------------------+------------------------------------------------
                var(Residual) |   .0360828     .00178      .0327575    .0397458
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 397.38        Prob >= chibar2 = 0.0000
```

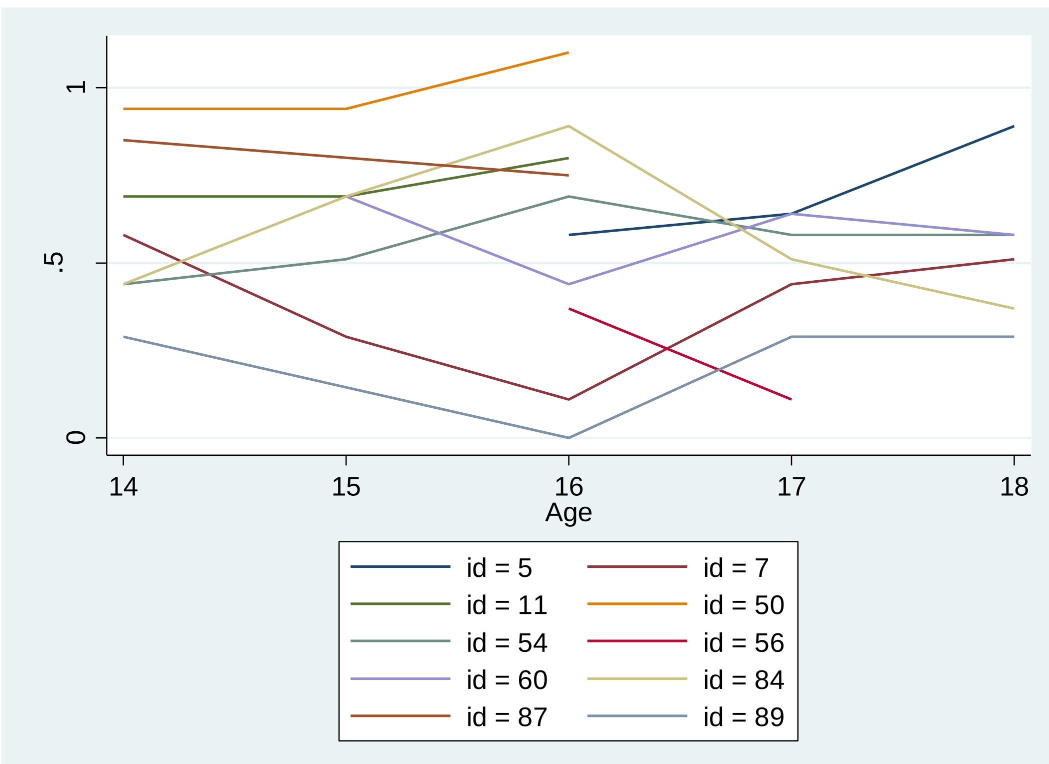
Let's examine time trends graphically:

```
. xtline attit
```


```
. xtline attit if id<100
```




```
. xtline attit if id<100, overlay
```



Very often, in this type of analysis, we are interested in understanding why and how the trajectory over time varies across units (that is why these models are also called growth curve models), so we want to explore that variation – that requires estimating a mixed effects model; random effects model cannot assess variation in the slope of age.

```
. mixed attit age || id: age, cov(unstructured)

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(1)      =      36.73
Log likelihood =  57.442108                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0323534   .0053383     6.06   0.000     .0218905    .0428164
       _cons |  -.0243373   .0870451    -0.28   0.780    -.1949426    .1462679
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    var(age) |   .0031015   .0006365      .0020743    .0046372
                  var(_cons) |   .8692899   .1703095      .5921053    1.276234
              cov(age,_cons) |  -.0505552   .0103397     -.0708206   -.0302899
-----------------------------+------------------------------------------------
               var(Residual) |   .0287285   .0016527      .0256652    .0321575
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 438.92                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Note that we specified covariance option – that is because we want to allow random effects to correlate with each other; if we do not, that would be too restrictive since usually random effects for intercepts and slopes are correlated. So we have two random effects now:

$$\begin{pmatrix} u_{0i} \\ u_{1i} \end{pmatrix} \sim N\left(0, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right)$$

Our tau matrix now contains the variance in the level-1 intercepts (τ_{00}), the variance in level-1 slopes (τ_{11}), as well as the covariance between level-1 intercepts and slopes ($\tau_{01} = \tau_{10}$). (This covariance is presented as a correlation in our output.) Note that covariance value indicates how much intercepts and slopes covary: in our example, there is a negative correlation between intercepts and slopes. That is, the higher the intercept, the smaller the slope (i.e. if the starting point in terms of deviant attitudes is higher, then the slope is less steep). We can see this as a variance-covariance matrix:

```
. estat recov

Random-effects covariance matrix for level id
             |       age      _cons
-------------+----------------------
         age |  .0031015
       _cons | -.0505552   .8692899
```

So far we assumed that the time trend is linear but the graph above shows that for many people it is not. Let's estimate a model with a quadratic trend.

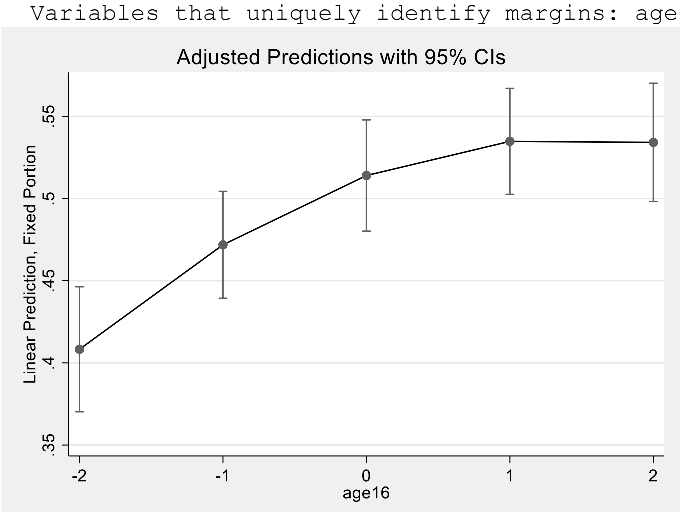
```
. tab age

        Age |      Freq.     Percent        Cum.
------------+-----------------------------------
         14 |        241       20.00       20.00
         15 |        241       20.00       40.00
         16 |        241       20.00       60.00
         17 |        241       20.00       80.00
         18 |        241       20.00      100.00
------------+-----------------------------------
      Total |      1,205      100.00

. gen age16=age-16
```

Note that the intercept will now correspond to value at age 16 rather than at the start of the study.

```
. mixed attit c.age16##c.age16 || id: c.age16##c.age16, cov(unstructured)
Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(2)      =      41.54
Log likelihood =  76.206955                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0314681   .0053202     5.91   0.000     .0210407    .0418956
             |
     c.age16#|
     c.age16 |  -.0106942   .0036435    -2.94   0.003    -.0178353   -.0035532
             |
       _cons |   .5140137   .0172699    29.76   0.000     .4801654     .547862
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0036617   .0006295      .0026143    .0051287
            var(age16#age16) |   .0011685   .0003037      .0007021    .0019447
                  var(_cons) |   .0579519    .006591      .0463722    .0724232
      cov(age16,age16#age16) |  -.0003337   .0002893     -.0009008    .0002333
            cov(age16,_cons) |  -.0003278   .0014214     -.0031136    .0024581
      cov(age16#age16,_cons) |   -.004129   .0011176     -.0063195   -.0019385
-----------------------------+------------------------------------------------
               var(Residual) |   .0229085   .0016112      .0199586    .0262943
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 471.08                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. margins, at(age16=(-2(1)2))

Adjusted predictions                            Number of obs     =      1,066

Expression   : Linear prediction, fixed portion, predict()
```

```
1._at        : age16          =          -2
2._at        : age16          =          -1
3._at        : age16          =           0
4._at        : age16          =           1
5._at        : age16          =           2
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   .4083006   .0194029    21.04   0.000     .3702716    .4463297
          2  |   .4718514   .0165959    28.43   0.000     .4393239    .5043788
          3  |   .5140137   .0172699    29.76   0.000     .4801654     .547862
          4  |   .5347876   .0164525    32.50   0.000     .5025413     .567034
          5  |   .5341731   .0183595    29.10   0.000     .4981892     .570157
------------------------------------------------------------------------------
```

```
. marginsplot, x(age16)

  Variables that uniquely identify margins: age16
```

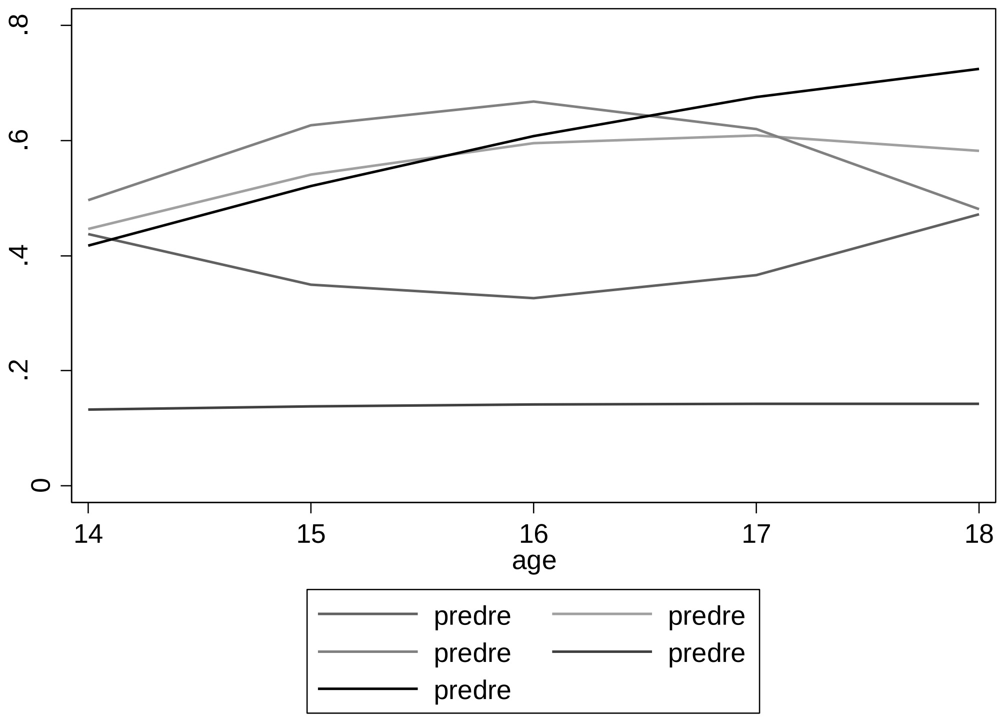


This is identical to calculating:

```
. gen pred= .5140183+.0314627 *age16 -.0106962 *age16sq
```

This is the average trajectory; let's see some of the variation across individuals, however. For that, we will obtain estimates of random effects for all three components of the equation and add them to the average coefficients:

```
. predict re*, reffects

. gen predre=.5140183+re3+(.0314627+re1) *age16 +(-.0106962+re2) *age16sq

. graph twoway (line predre age if id==7) (line predre age if id==54) (line predre age
if id==84)  (line predre age if id==104)  (line predre age if id==111)
```




```
. est store squared

. estat ic
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
     squared |      1,066          .   76.20696      10  -132.4139  -82.69722
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. qui mixed attit age16 || id: age16, cov(unstructured)

. est store linear

. estat ic
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
      linear |      1,066          .   57.44211       6  -102.8842   -73.0542
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. lrtest squared linear

Likelihood-ratio test                                 LR chi2(4)  =     37.53
(Assumption: linear nested in squared)                Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on
      the boundary of the parameter space.  If this is not true, then the
      reported test is conservative.
```

Both LR test and difference in BIC (almost 10) indicate that the model with age squared offers a better fit.

If we wanted to just test whether each variance component is significant, we would run LR tests, e.g. to test if the squared age slope variance is significant:

```
. qui mixed attit c.age16##c.age16  || id: c.age16##c.age16, cov(unstructured)

. . est store full

. qui mixed attit c.age16##c.age16  || id: c.age16, cov(unstructured)

. . lrtest . full

Likelihood-ratio test                                 LR chi2(3)  =     26.57
(Assumption: . nested in full)                        Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space.  If this is not true, then the reported test is
conservative.
```

We could also use this approach test whether the random intercept variance is statistically significant.

```
. qui mixed attit c.age16##c.age16  || id:

. est store re

. qui mixed attit c.age16##c.age16

. lrtest . re

Likelihood-ratio test                                 LR chi2(1)  =    400.75
(Assumption: . nested in re)                          Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space.  If
      this is not true, then the reported test is conservative.
```

Next, let’s add variables that could explain variation in attitudes. We start with time-varying (level 1) variables – here we have expo. But it is possible for effects of this variable to also vary across individuals so we allow for such variation:

```
. mixed attit age16 age16sq  expo  || id: age16 age16sq expo, cov(unstructured)

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log restricted-likelihood =  191.16984
Iteration 1:   log restricted-likelihood =  191.56353
Iteration 2:   log restricted-likelihood =  191.56453
Iteration 3:   log restricted-likelihood =  191.56453

Computing standard errors:

Mixed-effects REML regression                   Number of obs      =      1066
Group variable: id                              Number of groups   =       241

                                                Obs per group: min =         1
```

```
                                                                  avg =        4.4
                                                                  max =          5

                                                Wald chi2(3)       =     269.83
Log restricted-likelihood =  191.56453          Prob > chi2        =     0.0000

------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0229438   .0048663     4.71   0.000     .0134061    .0324816
     age16sq |  -.0045771   .0032443    -1.41   0.158    -.0109357    .0017816
        expo |   .4392177   .0303382    14.48   0.000     .3797559    .4986794
       _cons |   .2522643   .0215611    11.70   0.000     .2100052    .2945234
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  sd(age16) |    .051728   .0051497      .0425584    .0628733
                sd(age16sq) |   .0261037   .0047828      .0182282    .0373819
                   sd(expo) |     .23522   .0366039      .1733861    .3191056
                  sd(_cons) |   .2071172   .0215611      .1688905    .2539962
         corr(age16,age16sq) |  -.2236336   .1771147     -.5319718    .1370675
            corr(age16,expo) |   -.183421   .1671742     -.4812296    .1523468
           corr(age16,_cons) |   .1768226   .1443242     -.1128169    .4387654
          corr(age16sq,expo) |   .1805575   .2097605     -.2377791    .5423907
         corr(age16sq,_cons) |  -.4224947   .1591735     -.6807377   -.0708434
            corr(expo,_cons) |  -.6382323   .0978611     -.7927606   -.4066177
-----------------------------+------------------------------------------------
               sd(Residual) |   .1410003   .0053562      .1308836     .151899
------------------------------------------------------------------------------
LR test vs. linear regression:      chi2(10) =   290.82   Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

There is significant variation in slopes of all of these three level 1 variables. Next, we add level 2 (time invariant) variables as predictors of attitudes (but not yet of slopes). We have the following level 2 predictors: female, minority, and income.

```
. mixed attit c.age16##c.age16  expo  female minority income || id: c.age16##c.age16
expo, cov(unstructured)

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(6)      =     294.71
Log likelihood =  213.47687                     Prob > chi2       =     0.0000
---------------------------------------------------------------------------------
          attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
----------------+----------------------------------------------------------------
          age16 |   .0227999   .0048543     4.70   0.000     .0132857    .0323141
                |
c.age16#c.age16 |  -.0043997   .0032383    -1.36   0.174    -.0107467    .0019472
                |
           expo |   .4427109   .0298912    14.81   0.000     .3841253    .5012965
```

```
      female |  -.0497872   .0217828    -2.29   0.022    -.0924807   -.0070937
    minority |   .0224325   .0275622     0.81   0.416    -.0315885    .0764534
      income |   .0141915   .0048076     2.95   0.003     .0047688    .0236142
       _cons |   .2076963   .0330908     6.28   0.000     .1428396    .2725531
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0026491   .0005294      .0017906    .0039192
            var(age16#age16) |   .0006767   .0002489      .0003291    .0013914
                   var(expo) |   .0523096   .0165384      .0281489    .0972077
                  var(_cons) |    .038094   .0084247      .0246949    .0587631
      cov(age16,age16#age16) |  -.0003033   .0002387     -.0007712    .0001645
             cov(age16,expo) |  -.0020604   .0020852     -.0061474    .0020266
            cov(age16,_cons) |   .0020591   .0015392     -.0009577    .0050758
       cov(age16#age16,expo) |   .0011388   .0013086       -.001426    .0037037
      cov(age16#age16,_cons) |  -.0022439   .0011297       -.004458   -.0000297
             cov(expo,_cons) |  -.0274701   .0107463     -.0485325   -.0064077
-----------------------------+------------------------------------------------
               var(Residual) |   .0199329   .0015127      .0171781    .0231296
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 270.95            Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Since we are now trying to model variance in the constant (intercept), we should make sure that intercept meaningful by making 0 a meaningful value on all predictors. Dummies are ok as long as they are coded 0/1 but continuous predictors should be mean-centered.

```
. for var expo income: sum X \ gen Xm=X-r(mean)

->  sum expo

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        expo |      1066    .5601501    .3106114          0       1.61

->  gen expom=expo-r(mean)
(139 missing values generated)

->  sum income

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      income |      1205    4.091286    2.346617          1         10

->  gen incomem=income-r(mean)

. mixed attit c.age16##c.age16 expom female minority incomem  || id: c.age16##c.age16
expom, cov(unstructured)
Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(6)      =     294.71
Log likelihood =  213.47687                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
```

```
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0227999   .0048543     4.70   0.000     .0132857    .0323141
             |
    c.age16#|
     c.age16 |  -.0043997   .0032383    -1.36   0.174    -.0107467    .0019472
             |
       expom |   .4427109   .0298912    14.81   0.000     .3841253    .5012965
      female |  -.0497872   .0217828    -2.29   0.022    -.0924807   -.0070937
    minority |   .0224325   .0275622     0.81   0.416    -.0315885    .0764534
     incomem |   .0141915   .0048076     2.95   0.003     .0047688    .0236142
       _cons |   .5146037   .0176118    29.22   0.000     .4800851    .5491223
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0026491   .0005294      .0017906    .0039192
            var(age16#age16) |   .0006767   .0002489      .0003291    .0013914
                  var(expom) |   .0523096   .0165384      .0281489    .0972077
                  var(_cons) |   .0237323   .0036756      .0175189    .0321494
      cov(age16,age16#age16) |  -.0003033   .0002387     -.0007712    .0001645
            cov(age16,expom) |  -.0020604   .0020852     -.0061474    .0020266
            cov(age16,_cons) |   .0009049   .0009612     -.0009789    .0027888
      cov(age16#age16,expom) |   .0011388   .0013086      -.001426    .0037037
      cov(age16#age16,_cons) |  -.0016059   .0007489     -.0030738   -.0001381
            cov(expom,_cons) |   .0018312   .0055434     -.0090338    .0126961
-----------------------------+------------------------------------------------
               var(Residual) |   .0199329   .0015127      .0171781    .0231296
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 270.95             Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

The kind of centering we just applied is called grand-mean centering. The centering issue is important in mixed models.

Centering choices for time-varying (level-1) predictors:

<u>1. Natural metric (X):</u>
You should only use the original metric if the value of 0 for a predictor is a meaningful value. When 0 is not meaningful, the estimate of the intercept will be arbitrary and may be estimated with poor precision. Lack of precision in mixed models can be very problematic. First, because you are estimating within-group intercepts, thus with possibly small N, the estimates may be quite unstable. Second, because you may be trying to model variation in these intercepts, your model will be affected by the unreliability of the estimates.

<u>2. Grand-mean centering (X - grand mean):</u>
This will address the problems with estimation of intercept in original metric. Because the 0 values will fall in the middle of the distribution of the predictors, the intercept estimates will be estimated with much more precision. The intercept is also interpretable. Specifically, it will represent the value for a person with a (grand) average on every predictor. The interpretation of the intercepts is now "adjusted group mean." The interpretation of slopes does not change. So we can interpret the fixed effect for the intercept as the average attitudes value adjusted for exposure – i.e., the average attitudes level for someone with average exposure to deviant peers.

Note that while it may seem inappropriate at first to center a dummy variable, in mixed models it can actually is quite useful. If uncentered, the intercept in a model with a dummy variable is the average value when the dummy variable is 0. If the dummy variable is centered, the intercept then becomes the mean adjusted for the proportion of time points with the dummy variable=1, so essentially it is the mean for an average case. We would only consider centering dummy variables when we would like to treat them as controls rather than main predictors of interest.

3. Group-mean centering (X – group mean):
Predictors can also be centered around the mean value for a given person (averaged over time). Recall how we used group-mean centered variables to indicate the change component within random effects models along with group means to indicate cross-sectional effects of differences across individuals. The intercept can then be interpreted as the average outcome for each person. This allows interpretation of parameter estimates as effects of change over time within-person. Under grand-mean centering or no centering, the parameter estimates reflect a combination of change over time and differences across individuals. But when we use a group-centered predictor, we only estimate only change effects (within-person component). In order not to discard the effects of differences across individuals, we should include person level variables alongside group-mean centered predictors. This is a common way to separate within and between unit effects in mixed effects model (we did that in random effects model as well).

Level-2 predictors:

Centering issues for level-2 predictors are essentially the same issues faced in any regression. If the value of 0 for a predictor is not meaningful, the intercept will not have a meaningful interpretation and the estimate may lack precision. When these conditions exist, grand-mean centering is advisable.

Example of group-mean centering for our model:

```
. by id: egen expomean=mean(expo)

. gen expochange=expo-expomean
(139 missing values generated)

. sum expomean

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
    expomean |      1,066    .5601501    .2532099          0       1.32

. gen expomeanm=expomean-r(mean)

. mixed attit c.age16##c.age16 expochange expomeanm female minority incomem  || id:
c.age16##c.age16 expochange, cov(unstructured)
Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5
```

```
                                                Wald chi2(7)      =     384.26
Log likelihood =  227.43873                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |    .024962   .0049115     5.08   0.000     .0153357    .0345883
             |
     c.age16#|
     c.age16 |  -.0058366   .0032127    -1.82   0.069    -.0121333    .0004601
             |
  expochange |   .3470481   .0372215     9.32   0.000     .2740952     .420001
   expomeanm |   .6187336   .0407646    15.18   0.000     .5388365    .6986307
      female |  -.0433309   .0211836    -2.05   0.041    -.0848499   -.0018119
    minority |   .0118951   .0267071     0.45   0.656    -.0404499    .0642401
     incomem |   .0165267   .0046909     3.52   0.000     .0073327    .0257207
       _cons |    .520923   .0170787    30.50   0.000     .4874494    .5543966
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0027411   .0005396      .0018637    .0040317
            var(age16#age16) |   .0006593   .0002439      .0003193    .0013614
             var(expochange) |   .0688762   .0234179      .0353722     .134115
                  var(_cons) |   .0252743   .0034363       .019362    .0329919
      cov(age16,age16#age16) |  -.0003023   .0002421     -.0007768    .0001722
        cov(age16,expochange) |  -.0025692   .0025629     -.0075924     .002454
             cov(age16,_cons) |   .0012069   .0009569     -.0006687    .0030825
 cov(age16#age16,expochange) |   .0004807   .0015122     -.0024831    .0034446
      cov(age16#age16,_cons) |  -.0017197   .0007289     -.0031483   -.0002911
        cov(expochange,_cons) |    .006767   .0068674     -.0066929    .0202269
-----------------------------+------------------------------------------------
               var(Residual) |   .0191308   .0014619      .0164698    .0222217
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 272.29               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

We can compare that model to a model with grand-mean centered expo variable and level 2 average expomean variable:

```
. mixed attit c.age16##c.age16 expom expomean  female minority incomem || id:
c.age16##c.age16 expom, cov(unstructured)

Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(7)      =     335.83
Log likelihood =   225.9009                     Prob > chi2       =     0.0000
---------------------------------------------------------------------------------
          attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
----------------+----------------------------------------------------------------
          age16 |   .0252621   .0049018     5.15   0.000     .0156547    .0348695
                |
c.age16#c.age16 |  -.0057106   .0032313    -1.77   0.077    -.0120438    .0006225
                |
```

```
        expom |   .3426117   .0352018     9.73   0.000     .2736175     .411606
     expomean |   .2746784   .0534395     5.14   0.000     .1699389    .3794179
       female |  -.0424745   .0211061    -2.01   0.044    -.0838417   -.0011072
     minority |   .0159387   .0266649     0.60   0.550    -.0363235     .068201
      incomem |   .0154524   .0046555     3.32   0.001     .0063278    .0245771
        _cons |   .3631344   .0344126    10.55   0.000     .2956869     .430582
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(age16) |   .0027646   .0005339      .0018934    .0040367
           var(age16#age16) |   .0006896   .0002454      .0003433     .001385
                 var(expom) |   .0449557   .0153803      .0229916    .0879021
                 var(_cons) |   .0226649   .0034608      .0168028    .0305721
     cov(age16,age16#age16) |   -.000309   .0002378      -.000775     .000157
           cov(age16,expom) |  -.0014435   .0019936     -.0053509    .0024639
           cov(age16,_cons) |   .0014412    .000954     -.0004286     .003311
     cov(age16#age16,expom) |   .0007174   .0012469     -.0017266    .0031614
     cov(age16#age16,_cons) |  -.0017183   .0007347     -.0031582   -.0002784
           cov(expom,_cons) |   .0005616   .0051006     -.0094354    .0105585
-----------------------------+------------------------------------------------
              var(Residual) |   .0196105   .0014703      .0169305    .0227147
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 269.21               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Next, we will estimate a model where we will use cross-level interactions to explain variance in slopes across individuals. That is, we will introduce interactions of level 1 predictors with level 2 time-invariant variables and then see what happens to variance of slopes of those level 1 predictors.

```
. mixed attit c.age16##c.age16##c.expomeanm c.age16##c.age16##i.female
c.age16##c.age16##i.minority c.age16##c.age16##c.incomem c.expochange##c.expomeanm
c.expochange##i.female c.expochange##i.minority c.expochange##c.incomem  || id:
c.age16##c.age16 expochange, cov(unstructured)
Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5

                                                Wald chi2(19)     =     439.44
Log likelihood =  243.59055                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |    .023436   .0074107     3.16   0.002     .0089113    .0379608
             |
    c.age16#|
     c.age16 |  -.0127451   .0047738    -2.67   0.008    -.0221016   -.0033887
             |
   expomeanm |   .6235837     .04901    12.72   0.000     .5275259    .7196416
             |
    c.age16#|
 c.expomeanm |   -.045851   .0188732    -2.43   0.015    -.0828419   -.0088602
             |
    c.age16#|
```

```
     c.age16#|
 c.expomeanm |  -.0041202   .0126952    -0.32   0.746    -.0290023    .0207619
             |
       age16 |          0  (omitted)
    1.female |  -.0738721   .0252524    -2.93   0.003     -.123366   -.0243783
             |
      female#|
     c.age16 |
          1  |   .0048259   .0097991     0.49   0.622    -.0143799    .0240318
             |
      female#|
     c.age16#|
     c.age16 |
          1  |   .0148405   .0064114     2.31   0.021     .0022744    .0274066
             |
       age16 |          0  (omitted)
  1.minority |  -.0079024   .0318651    -0.25   0.804    -.0703567     .054552
             |
    minority#|
     c.age16 |
          1  |  -.0073877   .0124986    -0.59   0.554    -.0318845     .017109
             |
    minority#|
     c.age16#|
     c.age16 |
          1  |   .0047483   .0082399     0.58   0.564    -.0114016    .0208982
             |
       age16 |          0  (omitted)
     incomem |   .0102222   .0055722     1.83   0.067     -.000699    .0211435
             |
     c.age16#|
   c.incomem |  -.0024181   .0021411    -1.13   0.259    -.0066145    .0017784
             |
     c.age16#|
     c.age16#|
   c.incomem |   .0022079   .0013923     1.59   0.113    -.0005208    .0049367
             |
  expochange |   .4121574   .0551548     7.47   0.000     .3040559    .5202589
   expomeanm |          0  (omitted)
             |
          c. |
  expochange#|
 c.expomeanm |   .2282904    .156432     1.46   0.144    -.0783106    .5348915
             |
  expochange |          0  (omitted)
             |
      female#|
c.expochange |
          1  |  -.0151889   .0744531    -0.20   0.838    -.1611144    .1307366
             |
  expochange |          0  (omitted)
             |
    minority#|
c.expochange |
          1  |  -.3040498   .0908928    -3.35   0.001    -.4821965   -.1259031
             |
  expochange |          0  (omitted)
     incomem |          0  (omitted)
             |
          c. |
  expochange#|
   c.incomem |  -.0445934   .0168649    -2.64   0.008     -.077648   -.0115387
             |
```

```
       _cons |   .5369439   .0186606    28.77   0.000     .5003697     .573518
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0025617   .0005206      .0017201    .0038151
            var(age16#age16) |    .000551   .0002324      .0002411    .0012595
             var(expochange) |   .0545077   .0203652      .0262078    .1133664
                  var(_cons) |   .0247072   .0033559      .0189325    .0322433
      cov(age16,age16#age16) |  -.0003286   .0002313     -.0007819    .0001247
       cov(age16,expochange) |  -.0029839   .0024093     -.0077061    .0017383
            cov(age16,_cons) |   .0012685   .0009229     -.0005403    .0030773
 cov(age16#age16,expochange) |   .0013568   .0014238     -.0014339    .0041475
      cov(age16#age16,_cons) |  -.0014805   .0006986     -.0028496   -.0001113
       cov(expochange,_cons) |   .0017472   .0064933     -.0109795    .0144739
-----------------------------+------------------------------------------------
               var(Residual) |   .0191457   .0014546      .0164969    .0222199
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 278.01               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Let's simplify the model by omitting non-significant cross-level interactions; we will use LR test and BIC to make sure we do not omit anything important:

```
. est store full

. estat ic
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
        full |      1,066          .   243.5905      31  -425.1811  -271.0594
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. mixed attit c.age16##c.age16##c.expomeanm c.age16##c.age16##i.female
c.expochange##i.minority c.expochange##c.incomem  || id: c.age16##c.age16 expochange,
cov(unstructured)
Mixed-effects ML regression                     Number of obs     =      1,066
Group variable: id                              Number of groups  =        241

                                                Obs per group:
                                                              min =          1
                                                              avg =        4.4
                                                              max =          5
                                                Wald chi2(13)     =     430.09
Log likelihood =  240.81527                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
       attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       age16 |   .0219426   .0064814     3.39   0.001     .0092393    .0346459
             |
     c.age16#|
     c.age16 |  -.0122447   .0042228    -2.90   0.004    -.0205212   -.0039681
             |
   expomeanm |   .6318958    .049011    12.89   0.000      .535836    .7279556
             |
     c.age16#|
 c.expomeanm |  -.0408274   .0187166    -2.18   0.029    -.0775112   -.0041436
             |
     c.age16#|
```

```
     c.age16#|
 c.expomeanm |  -.0091057   .0124199    -0.73   0.463    -.0334482    .0152368
             |
       age16 |          0  (omitted)
    1.female |  -.0738328   .0252351    -2.93   0.003    -.1232926    -.024373
             |
      female#|
     c.age16 |
          1  |   .0039725   .0095473     0.42   0.677    -.0147399    .0226849
             |
      female#|
     c.age16#|
     c.age16 |
          1  |   .0151555   .0062652     2.42   0.016     .0028759    .0274351
             |
  expochange |   .4158301   .0420899     9.88   0.000     .3333353    .4983248
  1.minority |   .0038862   .0267865     0.15   0.885    -.0486143    .0563868
             |
    minority#|
c.expochange |
          1  |  -.3121521   .0882582    -3.54   0.000    -.4851349   -.1391693
             |
  expochange |          0  (omitted)
     incomem |   .0152012   .0047064     3.23   0.001     .0059768    .0244256
             |
          c. |
  expochange#|
   c.incomem |  -.0550901    .016154    -3.41   0.001    -.0867514   -.0234288
             |
       _cons |   .5347902   .0180973    29.55   0.000     .4993201    .5702604
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                  var(age16) |   .0026167   .0005263      .0017642     .003881
            var(age16#age16) |   .0005755   .0002339      .0002595    .0012763
             var(expochange) |   .0574034   .0207247      .0282894    .1164801
                  var(_cons) |    .024968   .0033839      .0191435    .0325648
      cov(age16,age16#age16) |  -.0003702   .0002346       -.00083    .0000897
       cov(age16,expochange) |  -.0030557   .0024417     -.0078412    .0017299
            cov(age16,_cons) |   .0013106   .0009329     -.0005178     .003139
 cov(age16#age16,expochange) |   .0015322   .0014623     -.0013339    .0043982
      cov(age16#age16,_cons) |  -.0015776   .0007069     -.0029631    -.000192
       cov(expochange,_cons) |   .0017044   .0066061     -.0112434    .0146521
-----------------------------+------------------------------------------------
               var(Residual) |   .0191298   .0014506      .0164878     .022195
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 278.16               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. est store reduced
. lrtest reduced full
Likelihood-ratio test                                 LR chi2(6)  =      5.55
(Assumption: . nested in full)                        Prob > chi2 =    0.4754

. estat ic
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |      1,066          .   240.8153      25   -431.6305  -307.3388
```

```
------------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.
```

No significant difference in model fit indicated by LR test, and BIC is substantially smaller in the reduced model; therefore, we can use the reduced model.

To summarize model building in mixed effects models, we have a number of options:

- The effects of level 1 predictors can be estimated as either fixed effects or random effects
- Level 2 predictors can be used to predict the intercept (i.e., as direct predictors of DV)
- Level 2 predictors can explain the variation in slopes of level 1 predictors (i.e., as cross-level interactions)

Because so many components are involved, it is best to proceed incrementally.

1. Start by fitting a model with only the time variable. Evaluate level 2 variance in intercepts and time slopes to see if a mixed effects model is necessary.
2. Estimate a model with random intercept and slopes using only level 1 variables (all slopes should be random effects). Evaluate slope variance and decide whether some slopes should be fixed (i.e., no random component included for it).
3. Estimate a model with both level 1 variables and level 2 variables used as predictors of intercepts.
4. For slopes with significant variance, use level 2 predictors to explain that variance (i.e., estimate a model with cross-level interactions).
5. If the slope variance remaining after entering level 2 predictors is not statistically significant, estimate that slope as non-randomly varying (i.e., keep cross-level interactions but do not include a random component for that slope).
6. When making decisions what variables to include and whether to estimate random or fixed effects, use LR tests and BIC values to select a model with best fit and parsimony.