SOCY7708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: Interpreting Interactions in HLM

So far, we looked at a cross-level interaction of a continuous level-1 variable, SES, and a dichotomous level 2 variable, SECTOR, but there can be various other combinations; we will discuss four:

(1) Two dichotomous variables
(2) A dichotomy and a continuous variable
(3) A multicategory variable and a continuous variable
(4) Two continuous variables

What is crucial for interpretation in all of these cases is to consider the size and significance of what's called simple slopes – that is, the slope of X when moderator Z is at a specified value. That means, we should always consider one variable as the focal one (X) and one as the moderator (Z). It's arbitrary for a given interaction which one is which – but it usually makes substantive sense to designate one variable as the focal one and the other as the moderator. In HLM, we typically discuss explaining variance in level 1 slopes using level 2 variables, so that would suggest that level 1 variable would serve as your focal variable, and level 2 variable serve as a moderator, but we can always invert that interpretation – for example, rather than discussing how the effect of SES is different in Catholic vs public schools (ses=focal, sector=moderator), we could discuss the size of the school type effect for students with different levels of SES – for instance, high, medium, low (sector=focal, ses=moderator). In what follows, I will discuss interpretation for each of the four possible variable combinations and show some tools for easier interpretation.

Example 1: Two dichotomous variables

Here, we will look at a cross-level interaction of sector on level 2 and minority on level 1:

```
. mixed mathach i.minority##i.sector ses size || id:minority, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(5)      =     879.58
Log likelihood = -23175.932                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
----------------+-------------------------------------------------------------
     1.minority |  -4.226579   .3040739   -13.90   0.000    -4.822553   -3.630605
       1.sector |   2.266543   .3311193     6.85   0.000     1.617561    2.915525
                |
minority#sector |
           1 1  |   2.187448   .4225161     5.18   0.000     1.359331    3.015564
                |
            ses |    2.04651   .1056623    19.37   0.000     1.839416    2.253605
           size |   .0009792   .0002577     3.80   0.000      .000474    .0014844
          _cons |   11.44099   .3914482    29.23   0.000     10.67376    12.20821
```

```
--------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
              var(minority) |   .5397437   .6465572      .0515865    5.647276
                 var(_cons) |   2.319074   .4180191      1.628858    3.301763
        cov(minority,_cons) |  -.0819745   .4043974     -.8745789    .7106299
-----------------------------+------------------------------------------------
              var(Residual) |   35.93477   .6106514      34.75763    37.15178
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 205.32                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

<u>Minority as the focal variable, sector as the moderator:</u>

For public schools, minority students have math achievement scores that are 4.2 units lower than non-minority students. For Catholic schools, minority students have math achievement scores that are 2 units lower (-4.2+2.2=-2) than non-minority students. So the effect of being a minority is more pronounced in public than in Catholic schools, and difference in effects is statistically significant (as indicated by the significant interaction term). We know that the effect of being a minority is significant for public schools, but what about in Catholic schools? We can find out if we reverse the dummy variable (by running the same mixed command with ib1.minority) or using margins command:

```
. margins, dydx(minority) at(sector=(0 1)) atmeans

Conditional marginal effects                             Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  1.minority
1._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       sector     =        0
       ses        = .0001434 (mean)
       size       = 1056.862 (mean)
2._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       sector     =        1
       ses        = .0001434 (mean)
       size       = 1056.862 (mean)
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
0.minority   |  (base outcome)
-------------+----------------------------------------------------------------
1.minority   |
         _at |
          1  |  -4.226579   .3040739   -13.90   0.000    -4.822553   -3.630605
          2  |  -2.039132    .296945    -6.87   0.000    -2.621133    -1.45713
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Here, we can see that even though the effect of being a minority is smaller in Catholic schools than in public schools, it is nevertheless significant in both types of schools.

Sector as the focal variable, minority as the moderator:

Among non-minority students, those in Catholic schools have math achievement scores that are 2.3 units higher than those in public schools. And among minority students, that gap is even higher – minority students in Catholic schools have scores that are 4.5 units higher than minority students in public schools. Again, this difference in effects of school type is significant because the interaction term is significant. And margins command will help us see if simple slopes of sector are significant at both values of minority variable:

```
. margins, dydx(sector) at(minority=(0 1)) atmeans

Conditional marginal effects                            Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  1.sector
1._at: minority =        0
       0.sector = .5068894 (mean)
       1.sector = .4931106 (mean)
       ses      = .0001434 (mean)
       size     = 1056.862 (mean)
2._at: minority =        1
       0.sector = .5068894 (mean)
       1.sector = .4931106 (mean)
       ses      = .0001434 (mean)
       size     = 1056.862 (mean)
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
0.sector     |  (base outcome)
-------------+----------------------------------------------------------------
1.sector     |
         _at |
          1  |   2.266543   .3311193     6.85   0.000     1.617561    2.915525
          2  |    4.45399   .4630216     9.62   0.000     3.546485    5.361496
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Here, we can see that indeed, the effect of school type is significant for both minority and non-minority students. Finally, to further assist our interpretation, we could calculate predicted values of math achievement for the 4 groups based on minority and sector variables:

```
. margins, at(minority=(0 1) sector=(0 1)) atmeans

Adjusted predictions                                    Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
1._at: minority =        0
       sector   =        0
       ses      = .0001434 (mean)
       size     = 1056.862 (mean)
2._at: minority =        0
       sector   =        1
       ses      = .0001434 (mean)
       size     = 1056.862 (mean)
3._at: minority =        1
       sector   =        0
       ses      = .0001434 (mean)
```

```
       size     = 1056.862 (mean)
4._at: minority =        1
       sector   =        1
       ses      = .0001434 (mean)
       size     = 1056.862 (mean)
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   12.47616    .213089    58.55   0.000     12.05851     12.8938
          2  |    14.7427   .2364074    62.36   0.000     14.27935    15.20605
          3  |   8.249577   .3181134    25.93   0.000     7.626086    8.873068
          4  |   12.70357   .3115603    40.77   0.000     12.09292    13.31421
------------------------------------------------------------------------------
```

The scores are highest for non-minority students in Catholic schools and lowest for minority students in public schools.

Example 2: A dichotomy and a continuous variable

Here, we will look at a dichotomous variable minority on level 1 and a continuous variable size on level 2, but similar interpretation approaches can be used if your continuous variable is on level 1 and your dichotomy is on level 2.

```
. sum size

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        size |      7,185    1056.862    604.1725        100       2713

. egen tagged=tag(id)

. sum size if tagged==1

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        size |        160    1097.825    629.5064        100       2713

. gen sizem=size-r(mean)

. mixed mathach i.minority##c.sizem || id: minority, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(3)      =     224.93
Log likelihood = -23393.211                     Prob > chi2       =     0.0000
----------------------------------------------------------------------------------
         mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+----------------------------------------------------------------
      1.minority |  -3.723111   .2577542   -14.44   0.000      -4.2283   -3.217922
           sizem |   .0002325   .0003452     0.67   0.501    -.0004442    .0009092
                 |
minority#c.sizem |
             1   |  -.0015012   .0004082    -3.68   0.000    -.0023012   -.0007012
                 |
           _cons |   13.69733    .209928    65.25   0.000     13.28588    14.10878
```

```
----------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
               var(minority) |   2.262594   .9714639      .9752954    5.249008
                  var(_cons) |   5.561104     .78949      4.210357     7.34519
        cov(minority,_cons) |   .9227327   .6520362     -.3552349      2.2007
-----------------------------+------------------------------------------------
               var(Residual) |   37.41324   .6361601      36.18693     38.6811
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 741.52                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

<u>Minority as the focal variable, size as the moderator</u>:

In schools of average (mean) size, minority students have math achievement scores that are 3.7 units lower than non-minority students. If school size is one SD (629.5) higher than average, minority students have math achievement scores that are 4.6 units lower than non-minority students (calculated as -3.7-.0015012*629.5).

```
. di -3.7 -.0015012*629.5
-4.6450054
```

If school size is one SD lower than average, minority students have math achievement scores that are 2.8 units lower than non-minority students.

```
. di -3.7 +.0015012*629.5
-2.7549946
```

So we conclude that the race/ethnicity gap in more pronounced in larger schools than in smaller schools, and that difference is statistically significant (as indicated by the significant interaction term). To see if simple slopes of minority are significant at various levels of size variable, we employ margins command:

```
. sum sizem if tagged==1

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
       sizem |        160   -3.86e-06    629.5064   -997.825   1615.175

. return list

scalars:
                  r(N) =  160
              r(sum_w) =  160
               r(mean) =  -3.86238098145e-06
                r(Var) =  396278.3592549189
                 r(sd) =  629.5064409955778
                r(min) =  -997.8250122070313
                r(max) =  1615.175048828125
                r(sum) =  -.0006179809570313

. global min=r(min)

. global max=r(max)
```

```
. global mean=r(mean)

. global plussd=r(mean)+r(sd)

. global minussd=r(mean)-r(sd)

. margins, dydx(minority) at(size=($min $minussd $mean $plussd $max)) atmeans

Conditional marginal effects                          Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  1.minority
1._at: 0.minority =   .725261 (mean)
       1.minority =   .274739 (mean)
       sizem      =  -997.825
2._at: 0.minority =   .725261 (mean)
       1.minority =   .274739 (mean)
       sizem      = -629.5064
3._at: 0.minority =   .725261 (mean)
       1.minority =   .274739 (mean)
       sizem      = -3.86e-06
4._at: 0.minority =   .725261 (mean)
       1.minority =   .274739 (mean)
       sizem      =  629.5064
5._at: 0.minority =   .725261 (mean)
       1.minority =   .274739 (mean)
       sizem      =  1615.175

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
0.minority   |  (base outcome)
-------------+----------------------------------------------------------------
1.minority   |
         _at |
          1  |  -2.225132   .4869101    -4.57   0.000    -3.179458   -1.270805
          2  |  -2.778068   .3680558    -7.55   0.000    -3.499444   -2.056692
          3  |  -3.723111   .2577542   -14.44   0.000      -4.2283   -3.217922
          4  |  -4.668154    .359792   -12.97   0.000    -5.373333   -3.962974
          5  |  -6.147883   .7023876    -8.75   0.000    -7.524537   -4.771229
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Size as the focal variable, minority as the moderator:

For non-minority students, if a school has one extra student (school size increases by 1), then their math achievement increases by .0002 of a unit, but that increase is not statistically significant. For minority students, if the school size increases by 1, their math achievement decreases by .001 of a unit:

```
. di .0002325-.0015012
-.0012687
```

But we don't know yet if that's a significant decrease – we need to use margins to look at that simple slope:

```
. margins, dydx(sizem) at(minority=(0 1)) atmeans

Conditional marginal effects                          Number of obs = 7,185
```

```
Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  sizem
1._at: minority =         0
       sizem    = -40.96321 (mean)
2._at: minority =         1
       sizem    = -40.96321 (mean)
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
sizem        |
         _at |
          1  |   .0002325   .0003452     0.67   0.501    -.0004442    .0009092
          2  |  -.0012687   .0004836    -2.62   0.009    -.0022166   -.0003209
------------------------------------------------------------------------------
```

Here, we can see that the effect of school size is statistically significant for minority students but not for the non-minority students. The size of coefficient for minority students still looks small, but that's because one unit for school size is small – it's one student. Let's look how much change that would be for one SD increase in school size:

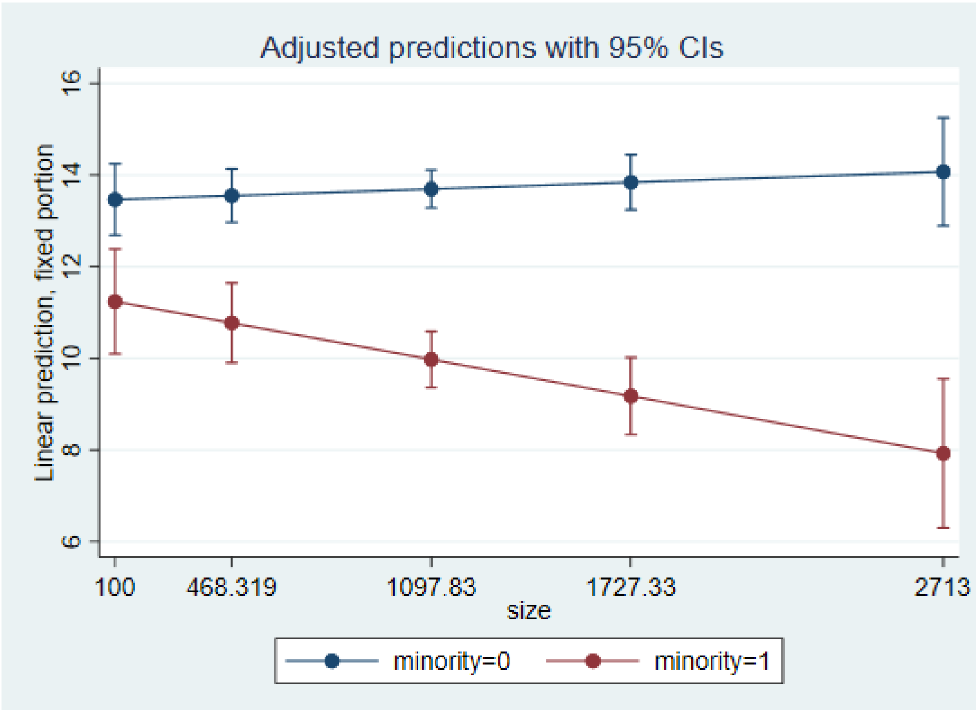
```
. di  -.0012687*629.5
-.79864665
```

We can also show these slopes on a graph, but to have a better scale, we will reestimate the model with size being uncentered (the results don't substantively change from that although the constant and main effects coefficients are less interpretable, but the graph is exactly the same, just labeled better):

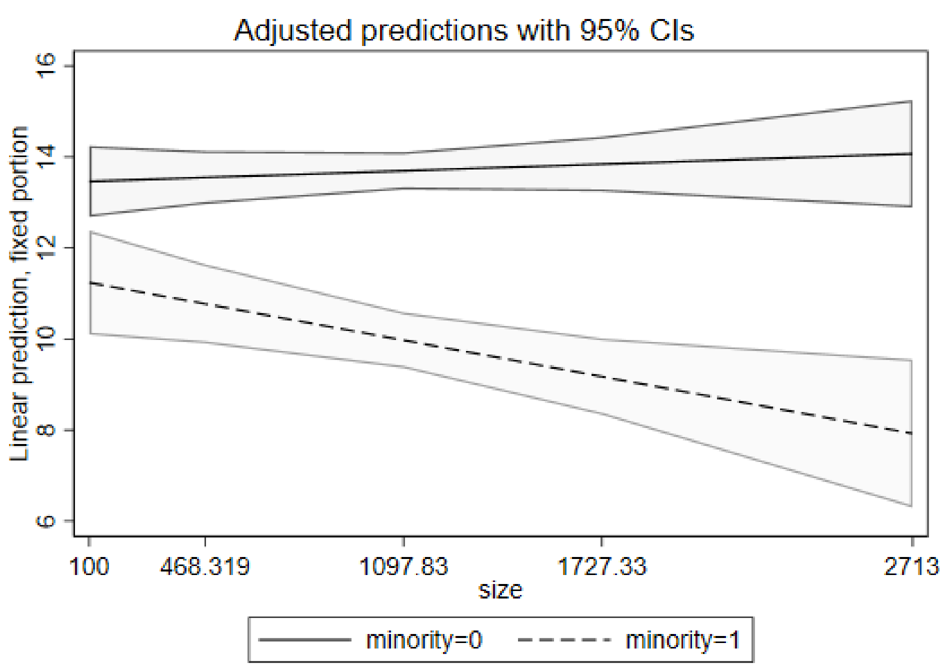
```
. mixed mathach i.minority##c.size || id: minority, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(3)      =     224.93
Log likelihood = -23393.211                     Prob > chi2       =     0.0000
---------------------------------------------------------------------------------
        mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
----------------+----------------------------------------------------------------
     1.minority |  -2.075007   .5219894    -3.98   0.000    -3.098088   -1.051927
           size |   .0002325   .0003452     0.67   0.501    -.0004442    .0009092
                |
minority#c.size |
             1  |  -.0015012   .0004082    -3.68   0.000    -.0023012   -.0007012
                |
          _cons |   13.44209   .4274142    31.45   0.000     12.60437     14.2798
---------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
               var(minority) |   2.262594   .9714639      .9752954    5.249008
                  var(_cons) |   5.561104     .78949      4.210357     7.34519
         cov(minority,_cons) |   .9227327   .6520362     -.3552349      2.2007
-----------------------------+------------------------------------------------
               var(Residual) |   37.41324   .6361601      36.18693     38.6811
```

```
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 741.52                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. sum size if tagged==1

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        size |        160    1097.825    629.5064        100       2713

. global min=r(min)

. global max=r(max)

. global mean=r(mean)

. global plussd=r(mean)+r(sd)

. global minussd=r(mean)-r(sd)

. margins, at(minority=(0 1) size=($min $minussd $mean $plussd $max))

Adjusted predictions                                     Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
1._at:  minority =        0
        size     =      100
2._at:  minority =        0
        size     = 468.3186
3._at:  minority =        0
        size     = 1097.825
4._at:  minority =        0
        size     = 1727.331
5._at:  minority =        0
        size     =     2713
6._at:  minority =        1
        size     =      100
7._at:  minority =        1
        size     = 468.3186
8._at:  minority =        1
        size     = 1097.825
9._at:  minority =        1
        size     = 1727.331
10._at: minority =        1
        size     =     2713

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   13.46534   .3976975    33.86   0.000     12.68586    14.24481
          2  |   13.55097    .297343    45.57   0.000     12.96819    14.13375
          3  |   13.69733    .209928    65.25   0.000     13.28588    14.10878
          4  |   13.84368   .3069108    45.11   0.000     13.24215    14.44522
          5  |   14.07285   .6020278    23.38   0.000     12.89289     15.2528
          6  |    11.2402   .5838319    19.25   0.000     10.09592    12.38449
          7  |    10.7729   .4436979    24.28   0.000     9.903269    11.64253
          8  |   9.974216   .3125363    31.91   0.000     9.361656    10.58678
          9  |    9.17553   .4287868    21.40   0.000     8.335124    10.01594
         10  |   7.924965   .8313537     9.53   0.000     6.295541    9.554388
------------------------------------------------------------------------------
```

```
. marginsplot, x(size)

Variables that uniquely identify margins: minority size
```

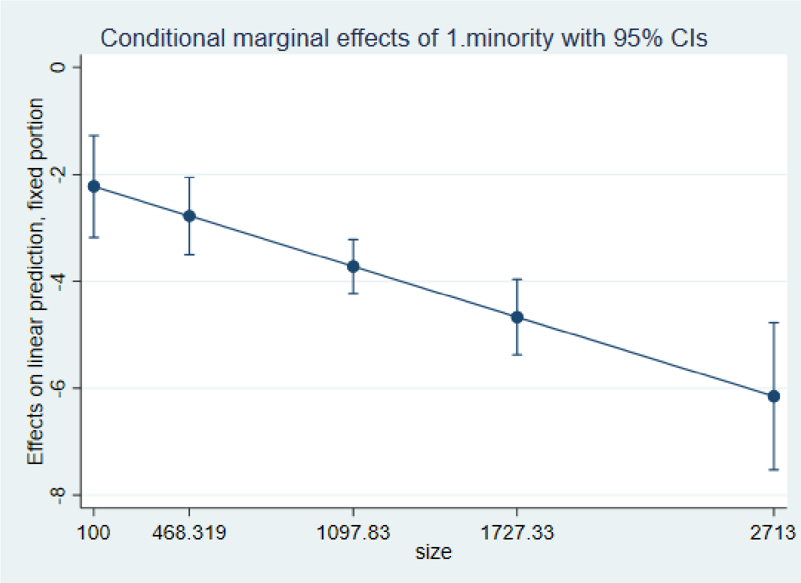



```
. marginsplot, x(size) plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5))
scheme(s1mono)

Variables that uniquely identify margins: minority size
```

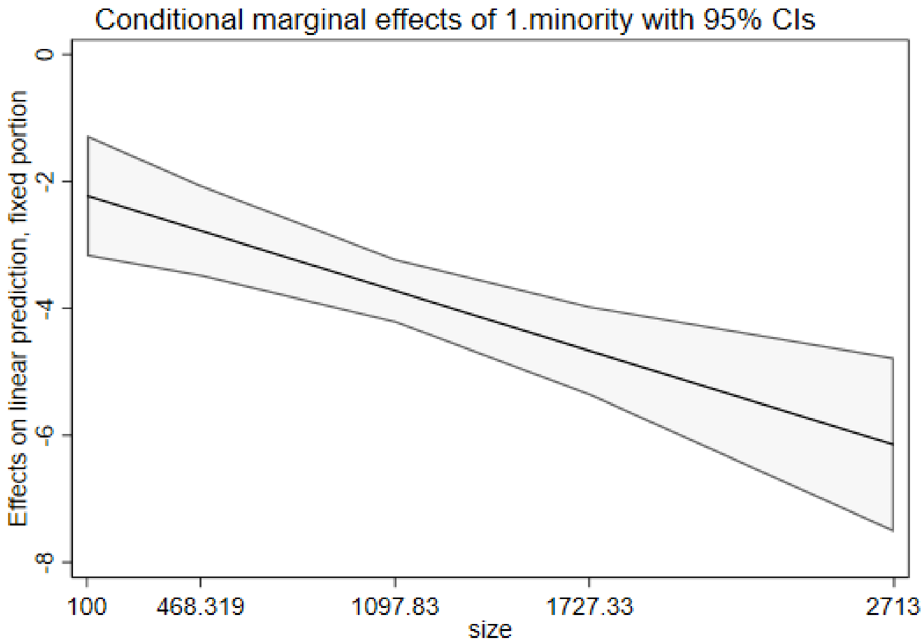


We can also graph the size of minority gap at different levels of school size – this graph would be used to assist interpretation when minority is the focal variable and school size is the moderator:

```
. margins, dydx(minority) at(size=($min $minussd $mean $plussd $max)) atmeans

Conditional marginal effects                          Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  1.minority
1._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       size       =      100
2._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       size       = 468.3186
3._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       size       = 1097.825
4._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       size       = 1727.331
5._at: 0.minority =  .725261 (mean)
       1.minority =  .274739 (mean)
       size       =     2713
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
0.minority   |  (base outcome)
-------------+----------------------------------------------------------------
1.minority   |
         _at |
          1  |  -2.225132   .4869101    -4.57   0.000    -3.179458   -1.270805
          2  |  -2.778068   .3680557    -7.55   0.000    -3.499444   -2.056692
          3  |  -3.723111   .2577542   -14.44   0.000      -4.2283   -3.217922
          4  |  -4.668154    .359792   -12.97   0.000    -5.373333   -3.962974
          5  |  -6.147883   .7023876    -8.75   0.000    -7.524537   -4.771229
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.

. marginsplot

Variables that uniquely identify margins: size
```




```
. marginsplot, plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5)) scheme(s1mono)

Variables that uniquely identify margins: size
```



Example 3: A multicategory variable and a continuous variable

For this example, I will again create dummy variables for school size but this time, I'll create four of them, approximately based on quartiles. I will use a function of egen variable to create that. My level 1 continuous variable will be SES.

```
. egen sized=cut(size), group(4)

. tab sized

      sized |      Freq.     Percent        Cum.
```

```
------------+-----------------------------------
          0 |      1,796       25.00       25.00
          1 |      1,764       24.55       49.55
          2 |      1,803       25.09       74.64
          3 |      1,822       25.36      100.00
------------+-----------------------------------
      Total |      7,185      100.00

. mixed mathach c.ses##i.sized || id:ses, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(7)      =     457.54
Log likelihood = -23308.899                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
     mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         ses |   2.100147   .2363191     8.89   0.000      1.63697    2.563324
             |
       sized |
          1  |   1.065607   .5351867     1.99   0.046       .01666    2.114553
          2  |   .9430464   .5394171     1.75   0.080    -.1141917    2.000284
          3  |  -.2007692   .5149239    -0.39   0.697    -1.210001    .8084631
             |
 sized#c.ses |
          1  |  -.0825774   .3329181    -0.25   0.804     -.735085    .5699301
          2  |   .3169989   .3335864     0.95   0.342    -.3368185    .9708163
          3  |   .8111932   .3194897     2.54   0.011      .185005    1.437381
             |
       _cons |   12.25458   .3819801    32.08   0.000     11.50592    13.00325
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    var(ses) |   .3036592    .218077      .0743163    1.240762
                  var(_cons) |   4.534295    .635098      3.445767    5.966693
              cov(ses,_cons) |  -.0604166   .2803227     -.6098389    .4890058
-----------------------------+------------------------------------------------
               var(Residual) |   36.80666   .6285147      35.59518    38.05937
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 428.23                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

First, I will test if the interaction terms between SES and SIZE dummies are jointly significant – you cannot judge their significance by individual coefficient significance tests because those will change depending on the omitted category.

```
. mat list e(b)

e(b)[1,14]
        mathach:     mathach:     mathach:     mathach:     mathach:     mathach:
mathach:
                          0b.           1.           2.           3.    0b.sized#
1.sized#
```

```
            ses        sized        sized        sized        sized       co.ses
c.ses
y1    2.1001473            0    1.0656066    .94304639   -.20076917            0   -
.08257741

        mathach:     mathach:     mathach:    lns1_1_1:    lns1_1_2:  atr1_1_1_2:
lnsig_e:
        2.sized#     3.sized#
          c.ses        c.ses        _cons        _cons        _cons        _cons
_cons
y1    .31699893    .81119323    12.254582   -.59592467    .75583481   -.05153382
1.8028394

. test 1.sized#c.ses=0

 ( 1)  [mathach]1.sized#c.ses = 0

           chi2(  1) =    0.06
         Prob > chi2 =    0.8041

. test 2.sized#c.ses=0, acc

 ( 1)  [mathach]1.sized#c.ses = 0
 ( 2)  [mathach]2.sized#c.ses = 0

           chi2(  2) =    1.61
         Prob > chi2 =    0.4475

. test 3.sized#c.ses=0, acc

 ( 1)  [mathach]1.sized#c.ses = 0
 ( 2)  [mathach]2.sized#c.ses = 0
 ( 3)  [mathach]3.sized#c.ses = 0

           chi2(  3) =    9.92
         Prob > chi2 =    0.0193
```

They are jointly significant. And in fact, if we wanted to better highlight that with significant coefficients when presenting the results, we may want to omit the last rather than the first SIZE category (so compare everything to the largest schools):

```
. mixed mathach c.ses##ib3.sized || id:ses, cov(unstr)

Computing standard errors ...

Mixed-effects ML regression                       Number of obs     =      7,185
Group variable: id                                Number of groups  =        160
                                                  Obs per group:
                                                                min =         14
                                                                avg =       44.9
                                                                max =         67
                                                  Wald chi2(7)      =     457.54
Log likelihood = -23308.899                       Prob > chi2       =     0.0000

------------------------------------------------------------------------------
     mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         ses |   2.911341   .2150045    13.54   0.000     2.489939    3.332742
             |
       sized |
         0   |   .2007692   .5149239     0.39   0.697    -.8084631    1.210001
         1   |   1.266376   .5096605     2.48   0.013     .2674596    2.265292
```

```
          2  |   1.143816    .514101     2.22   0.026     .1361962    2.151435
             |
 sized#c.ses |
          0  |  -.8111932   .3194897    -2.54   0.011    -1.437381    -.185005
          1  |  -.8937706   .3181426    -2.81   0.005    -1.517319   -.2702227
          2  |  -.4941943   .3188418    -1.55   0.121    -1.119113    .1307242
             |
       _cons |   12.05381   .3453083    34.91   0.000     11.37702    12.73061
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    var(ses) |   .3036591   .2180771      .0743163    1.240763
                  var(_cons) |   4.534295    .635098      3.445767    5.966693
               cov(ses,_cons) |  -.0604166   .2803227     -.6098389    .4890057
-----------------------------+------------------------------------------------
               var(Residual) |   36.80666   .6285147      35.59518    38.05937
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 428.23                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

<u>SES as the focal variable, SIZE dummies as a moderator:</u>

First, let's focus on level 1 variable – SES – as our focal one. The main effect for SES shows that for largest schools, 1 unit increase in SES translates into 2.9 units increase in math achievement. For the next largest size of schools, that effect would be 2.9-.5=2.4. Instead of calculating that manually, I will use margins again:

```
. margins, dydx(ses) at(sized=(0 1 2 3)) atmeans

Conditional marginal effects                             Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  ses
1._at: ses   = .0001434 (mean)
       sized =        0
2._at: ses   = .0001434 (mean)
       sized =        1
3._at: ses   = .0001434 (mean)
       sized =        2
4._at: ses   = .0001434 (mean)
       sized =        3

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ses          |
         _at |
          1  |   2.100147   .2363191     8.89   0.000      1.63697    2.563324
          2  |    2.01757   .2344947     8.60   0.000     1.557969    2.477171
          3  |   2.417146   .2354425    10.27   0.000     1.955687    2.878605
          4  |   2.911341   .2150045    13.54   0.000     2.489939    3.332742
------------------------------------------------------------------------------
```

The effect of SES is significant for all school sizes but it's more pronounced in the largest schools (the largest schools are significantly different from the smallest and second smallest, but not from the second largest category with regard to the size of SES effect on math achievement).

I may also want to estimate and present this model with an alternative parametrization that allows me to right away see SES slopes separately for each size category; this model doesn't include the interaction terms so we couldn't see if SES slopes are different depending on SIZE, so it should only be used as a follow-up to a standard model with interactions if we find that they are significant. This parametrization is called the separate slope parameterization, it includes four separate SES coefficients for the four SIZE groups rather than the more conventional interaction term parameterization (main effect of SES and three interaction terms for SIZE). For that, we create four variables in which one size-based group's SES values are included along with zeroes for the other three groups. These variables allow us to obtain separate simple slopes for SES for each size group. For more details on that approach, or to justify using it, see: Cohen, Jacob, Patricia Cohen, Stephen G. West, and Leona Aiken. 2003. Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences, 3rd ed. Mahwah, NJ: Lawrence Erlbaum.

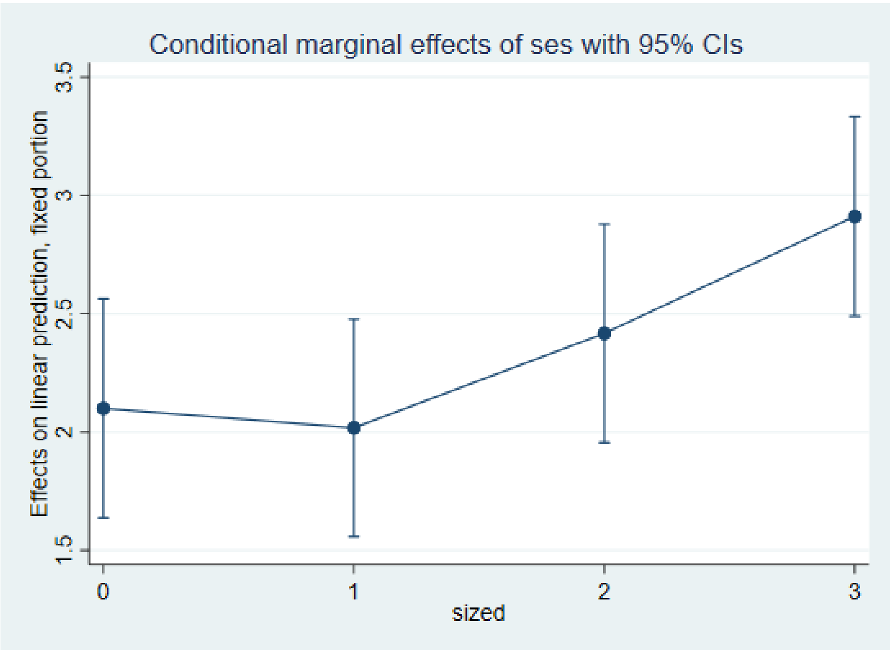
```
. tab sized, gen(sized_)

      sized |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      1,796       25.00       25.00
          1 |      1,764       24.55       49.55
          2 |      1,803       25.09       74.64
          3 |      1,822       25.36      100.00
------------+-----------------------------------
      Total |      7,185      100.00

. for num 1/4: gen ses_sized_X=ses*sized_X

->  gen ses_sized_1=ses*sized_1

->  gen ses_sized_2=ses*sized_2

->  gen ses_sized_3=ses*sized_3

->  gen ses_sized_4=ses*sized_4

. mixed mathach i.sized ses_sized* || id:ses, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(7)      =     457.54
Log likelihood = -23308.899                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
     mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       sized |
          1  |   1.065607   .5351867     1.99   0.046       .01666    2.114553
          2  |   .9430464   .5394171     1.75   0.080    -.1141917    2.000284
          3  |  -.2007692   .5149239    -0.39   0.697    -1.210001    .8084631
             |
 ses_sized_1 |   2.100147   .2363191     8.89   0.000      1.63697    2.563324
 ses_sized_2 |    2.01757   .2344947     8.60   0.000     1.557969    2.477171
 ses_sized_3 |   2.417146   .2354425    10.27   0.000     1.955687    2.878605
 ses_sized_4 |   2.911341   .2150045    13.54   0.000     2.489939    3.332742
```

```
       _cons |   12.25458   .3819801    32.08   0.000     11.50592    13.00325
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    var(ses) |   .3036591   .2180771      .0743162    1.240763
                  var(_cons) |   4.534295    .635098      3.445767    5.966693
              cov(ses,_cons) |  -.0604166   .2803227     -.6098389    .4890057
-----------------------------+------------------------------------------------
               var(Residual) |   36.80666   .6285147      35.59518    38.05937
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 428.23                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

I omitted the smallest school category, but perhaps omitting the largest could be better; that doesn't affect the simple slopes for SES that we can see in this model, however.

Next, we turn to graphic representation of our interactions results. Since my SIZED variable is ordinal, I could consider a graph illustrating the size of SES effect for each SIZE category:

```
. marginsplot

Variables that uniquely identify margins: sized
```



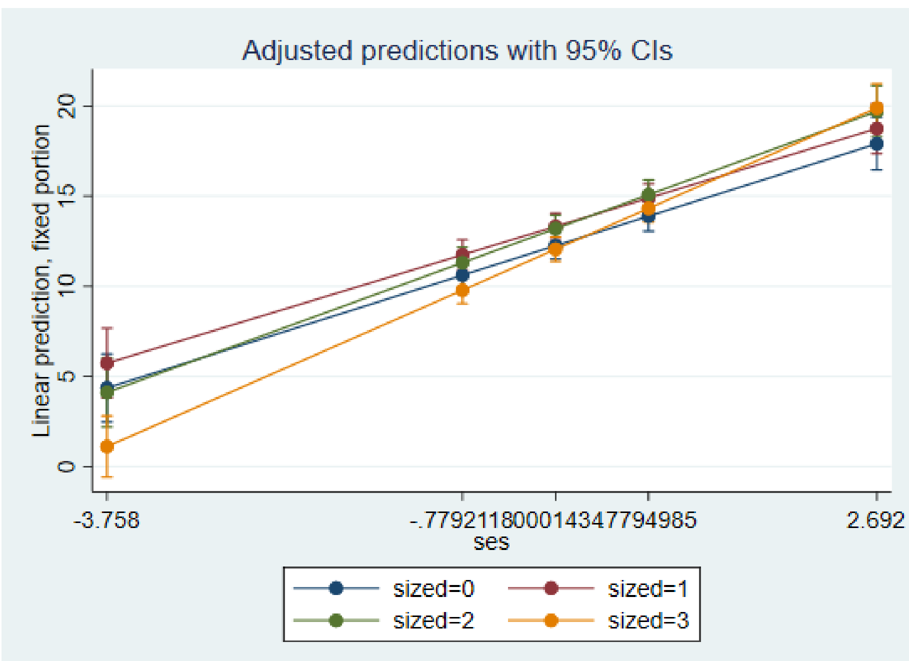
We could also illustrate this with a graph showing actual slopes of SES at each level of SIZE – for that, we do margins without dydx to generate predicted values:

```
. sum ses

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
         ses |      7,185    .0001434    .7793552     -3.758      2.692
```

```
. global min=r(min)

. global max=r(max)

. global mean=r(mean)

. global plussd=r(mean)+r(sd)

. global minussd=r(mean)-r(sd)

. margins, at(sized=(0 1 2 3) ses=($min $minussd $mean $plussd $max)) atmeans

Adjusted predictions                                     Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
1._at:  ses   =    -3.758
        sized =         0
2._at:  ses   =    -3.758
        sized =         1
3._at:  ses   =    -3.758
        sized =         2
4._at:  ses   =    -3.758
        sized =         3
5._at:  ses   = -.7792118
        sized =         0
6._at:  ses   = -.7792118
        sized =         1
7._at:  ses   = -.7792118
        sized =         2
8._at:  ses   = -.7792118
        sized =         3
9._at:  ses   =  .0001434
        sized =         0
10._at: ses   =  .0001434
        sized =         1
11._at: ses   =  .0001434
        sized =         2
12._at: ses   =  .0001434
        sized =         3
13._at: ses   =  .7794985
        sized =         0
14._at: ses   =  .7794985
        sized =         1
15._at: ses   =  .7794985
        sized =         2
16._at: ses   =  .7794985
        sized =         3
17._at: ses   =     2.692
        sized =         0
18._at: ses   =     2.692
        sized =         1
19._at: ses   =     2.692
        sized =         2
20._at: ses   =     2.692
        sized =         3

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   4.362229   .9643754     4.52   0.000     2.472088     6.25237
          2  |   5.738162    .984626     5.83   0.000      3.80833    7.667993
```

3	4.113994	.97967	4.20	0.000	2.193876	6.034112
4	1.112996	.8670579	1.28	0.199	-.5864064	2.812398
5	10.61812	.4229256	25.11	0.000	9.789204	11.44704
6	11.74807	.4298475	27.33	0.000	10.90559	12.59056
7	11.31416	.4304879	26.28	0.000	10.47042	12.1579
8	9.785262	.3782832	25.87	0.000	9.043841	10.52668
9	12.25488	.3819804	32.08	0.000	11.50622	13.00355
10	13.32048	.3748519	35.54	0.000	12.58578	14.05517
11	13.19798	.3808684	34.65	0.000	12.45149	13.94446
12	12.05423	.3453094	34.91	0.000	11.37744	12.73102
13	13.89164	.4251986	32.67	0.000	13.05827	14.72502
14	14.89288	.403804	36.88	0.000	14.10144	15.68432
15	15.08179	.4148992	36.35	0.000	14.2686	15.89498
16	14.3232	.389275	36.79	0.000	13.56023	15.08616
17	17.90818	.7442504	24.06	0.000	16.44947	19.36688
18	18.75149	.7081423	26.48	0.000	17.36355	20.13942
19	19.70459	.7238617	27.22	0.000	18.28584	21.12333
20	19.89114	.6846703	29.05	0.000	18.54921	21.23307

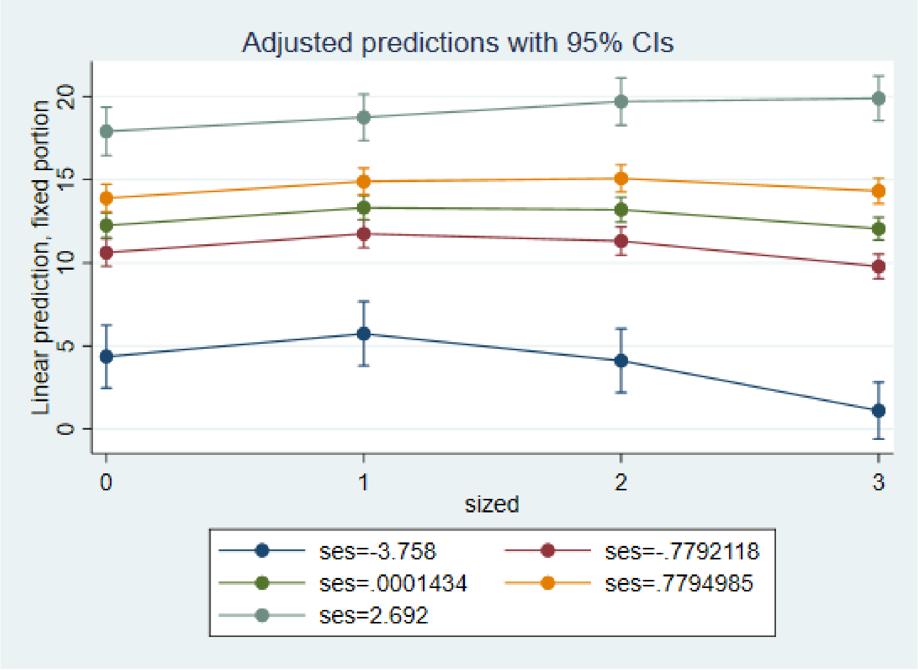
```
. marginsplot, x(ses)

Variables that uniquely identify margins: sized ses
```



<u>SIZE as focal, SES as a moderator:</u>

We can use the same predicted values to create a graph focusing on SIZE as focal:

```
. marginsplot, x(sized)

Variables that uniquely identify margins: sized ses
```



To examine which SIZE effects are statistically significant, we do margins with dydx focusing on sized:

```
. margins, dydx(sized) at(ses=($min $minussd $mean $plussd $max)) atmeans

Conditional marginal effects                          Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  0.sized 1.sized 2.sized
1._at: ses      =     -3.758
       0.sized =   .2499652  (mean)
       1.sized =   .2455115  (mean)
       2.sized =   .2509395  (mean)
       3.sized =   .2535839  (mean)
2._at: ses      = -.7792118
       0.sized =   .2499652  (mean)
       1.sized =   .2455115  (mean)
       2.sized =   .2509395  (mean)
       3.sized =   .2535839  (mean)
3._at: ses      =   .0001434
       0.sized =   .2499652  (mean)
       1.sized =   .2455115  (mean)
       2.sized =   .2509395  (mean)
       3.sized =   .2535839  (mean)
4._at: ses      =   .7794985
       0.sized =   .2499652  (mean)
       1.sized =   .2455115  (mean)
       2.sized =   .2509395  (mean)
       3.sized =   .2535839  (mean)
5._at: ses      =      2.692
       0.sized =   .2499652  (mean)
       1.sized =   .2455115  (mean)
       2.sized =   .2509395  (mean)
       3.sized =   .2535839  (mean)

------------------------------------------------------------------------------
             |            Delta-method
```

```
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
0.sized      |
         _at |
          1  |   3.249233   1.296846     2.51   0.012      .7074619    5.791005
          2  |   .8328605   .5674189     1.47   0.142     -.2792601    1.944981
          3  |   .2006529   .5149248     0.39   0.697     -.8085812    1.209887
          4  |  -.4315548   .5764797    -0.75   0.454     -1.561434    .6983247
          5  |  -1.982963   1.011277    -1.96   0.050      -3.96503   -.0008955
-------------+----------------------------------------------------------------
1.sized      |
         _at |
          1  |   4.625166   1.311975     3.53   0.000      2.053742    7.196589
          2  |   1.962812   .5725967     3.43   0.001      .8405435    3.085081
          3  |   1.266248   .5096593     2.48   0.013      .2673338    2.265161
          4  |   .5696828   .5608856     1.02   0.310     -.5296327    1.668998
          5  |  -1.139655   .9850071    -1.16   0.247     -3.070233    .7909237
-------------+----------------------------------------------------------------
2.sized      |
         _at |
          1  |   3.000998   1.308259     2.29   0.022      .4368563    5.565139
          2  |   1.528898   .5730777     2.67   0.008       .405686    2.652109
          3  |   1.143745   .5141005     2.22   0.026      .1361262    2.151363
          4  |   .7585918   .5689256     1.33   0.182     -.3564819    1.873665
          5  |  -.1865555    .996368    -0.19   0.851     -2.139401     1.76629
-------------+----------------------------------------------------------------
3.sized      |  (base outcome)
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Compared to largest schools, the smallest schools have higher math achievement scores for students with minimum SES (3.2 units higher); there are no significant differences between the largest and smallest schools at mean SES, maximum SES, or at mean+1SD and mean-1SD. The second and third category of school size are different from the largest schools in terms of math achievement for students whose SES is at mean or below (including mean itself, mean-1SD, and minimum SES). For minimum SES, the gap between the largest schools and second smallest is 4.6; the gap between the largest schools and second largest is 3 units. At mean SES, those gaps are 1.27 and 1.14, respectively. If we wanted to see those comparisons to another SIZE category, we could reestimate the model and the margins:

```
. mixed mathach c.ses##ib2.sized || id:ses, cov(unstr)

Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:   log likelihood = -23309.269
Iteration 1:   log likelihood = -23308.899
Iteration 2:   log likelihood = -23308.899

Computing standard errors ...

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(7)      =     457.54
Log likelihood = -23308.899                     Prob > chi2       =     0.0000
```

```
------------------------------------------------------------------------------
     mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         ses |   2.417146   .2354425    10.27   0.000     1.955687    2.878605
             |
       sized |
          0  |  -.9430464   .5394171    -1.75   0.080    -2.000284    .1141917
          1  |   .1225602    .534395     0.23   0.819    -.9248346    1.169955
          3  |  -1.143816    .514101    -2.22   0.026    -2.151435   -.1361962
             |
 sized#c.ses |
          0  |  -.3169989   .3335864    -0.95   0.342    -.9708163    .3368184
          1  |  -.3995763   .3322965    -1.20   0.229    -1.050865    .2517128
          3  |   .4941943   .3188418     1.55   0.121    -.1307242    1.119113
             |
       _cons |   13.19763     .38087    34.65   0.000     12.45114    13.94412
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                   var(ses) |   .3036591   .2180771      .0743162    1.240763
                 var(_cons) |   4.534295    .635098      3.445767    5.966693
            cov(ses,_cons) |  -.0604166   .2803227     -.6098389    .4890057
-----------------------------+------------------------------------------------
              var(Residual) |   36.80666   .6285147      35.59518    38.05937
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 428.23                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. margins, dydx(sized) at(ses=($min $minussd $mean $plussd $max)) atmeans

Conditional marginal effects                             Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  0.sized 1.sized 3.sized
1._at: ses     =    -3.758
       0.sized =  .2499652 (mean)
       1.sized =  .2455115 (mean)
       2.sized =  .2509395 (mean)
       3.sized =  .2535839 (mean)
2._at: ses     = -.7792118
       0.sized =  .2499652 (mean)
       1.sized =  .2455115 (mean)
       2.sized =  .2509395 (mean)
       3.sized =  .2535839 (mean)
3._at: ses     =  .0001434
       0.sized =  .2499652 (mean)
       1.sized =  .2455115 (mean)
       2.sized =  .2509395 (mean)
       3.sized =  .2535839 (mean)
4._at: ses     =  .7794985
       0.sized =  .2499652 (mean)
       1.sized =  .2455115 (mean)
       2.sized =  .2509395 (mean)
       3.sized =  .2535839 (mean)
5._at: ses     =     2.692
       0.sized =  .2499652 (mean)
       1.sized =  .2455115 (mean)
       2.sized =  .2509395 (mean)
       3.sized =  .2535839 (mean)
```

```
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
0.sized      |
         _at |
          1  |   .2482356    1.37469     0.18   0.857    -2.446108    2.942579
          2  |  -.6960371   .6034782    -1.15   0.249    -1.878833    .4867584
          3  |  -.9430918   .5394161    -1.75   0.080    -2.000328    .1141444
          4  |  -1.190147   .5940835    -2.00   0.045    -2.354529   -.0257643
          5  |  -1.796407   1.038212    -1.73   0.084    -3.831266    .2384509
-------------+----------------------------------------------------------------
1.sized      |
         _at |
          1  |   1.624168   1.388971     1.17   0.242    -1.098166    4.346502
          2  |   .4339148   .6083492     0.71   0.476    -.7584276    1.626257
          3  |   .1225029   .5343919     0.23   0.819     -.924886    1.169892
          4  |   -.188909   .5789637    -0.33   0.744    -1.323657     .945839
          5  |  -.9530993   1.012641    -0.94   0.347    -2.937839     1.03164
-------------+----------------------------------------------------------------
2.sized      |  (base outcome)
-------------+----------------------------------------------------------------
3.sized      |
         _at |
          1  |  -3.000998   1.308259    -2.29   0.022    -5.565139   -.4368563
          2  |  -1.528898   .5730777    -2.67   0.008    -2.652109    -.405686
          3  |  -1.143745   .5141005    -2.22   0.026    -2.151363   -.1361262
          4  |  -.7585918   .5689256    -1.33   0.182    -1.873665    .3564819
          5  |   .1865555    .996368     0.19   0.851     -1.76629    2.139401
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Example 4: Two continuous variables

Here, we will examine an interaction of SES and SIZE as a continuous variable rather than a set of dummies. We may want to use mean-centered SIZE variable here (we created it above).

```
. mixed mathach c.ses##c.sizem || id: ses, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(3)      =     438.94
Log likelihood =  -23314.32                     Prob > chi2       =     0.0000
-------------------------------------------------------------------------------
      mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------+----------------------------------------------------------------
          ses |   2.392577   .1153567    20.74   0.000     2.166482    2.618672
        sizem |   -.000266   .0003041    -0.87   0.382    -.0008621    .0003301
              |
c.ses#c.sizem |   .0004921   .0001853     2.65   0.008     .0001288    .0008554
              |
        _cons |   12.67515   .1890736    67.04   0.000     12.30458    13.04573
-------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
```

```
id: Unstructured             |
                   var(ses) |   .3168798    .221958      .0802919    1.250597
                 var(_cons) |   4.788075   .6640414       3.64847    6.283637
             cov(ses,_cons) |  -.1128863   .2887151     -.6787576    .4529849
-----------------------------+------------------------------------------------
              var(Residual) |   36.82078    .628957      35.60845    38.07438
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 461.36                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

<u>SES as the focal variable, SIZE as the moderator:</u>

The main effect of SES shows that at average school size, SES has a positive effect on math achievement – in average size schools, one unit increase in SES translates into 2.4 units increase in math achievement. That effect gets more pronounced in schools that are larger in size. Again, the unit for school size here is one student so to understand how much that effect is moderated, it is better to look at a standard deviation of sizem:

```
. sum sizem

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
       sizem |      7,185   -40.96321    604.1725   -997.825   1615.175

. global sizesd=r(sd)

. qui mixed mathach c.ses##c.sizem || id: ses, cov(unstr)

. mat list e(b)

e(b)[1,8]
        mathach:     mathach:     mathach:     mathach:    lns1_1_1:
                                    c.ses#
            ses        sizem      c.sizem        _cons        _cons
y1    2.3925766   -.00026597    .00049209    12.675153   -.57461631

       lns1_1_2:  atr1_1_1_2:     lnsig_e:
                                                    
          _cons        _cons        _cons
y1    .78306421    -.0919039    1.8030312

. di e(b)[1,1]+e(b)[1,3]*$sizesd
2.6898841
```

So for a school that’s one SD above the average size, one unit increase in SES translates into 2.7 units increase in math achievement. And for a school that’s one SD below the average size, one unit increase in SES translates into only 2.1 units increase in math achievement, as per this calculation:

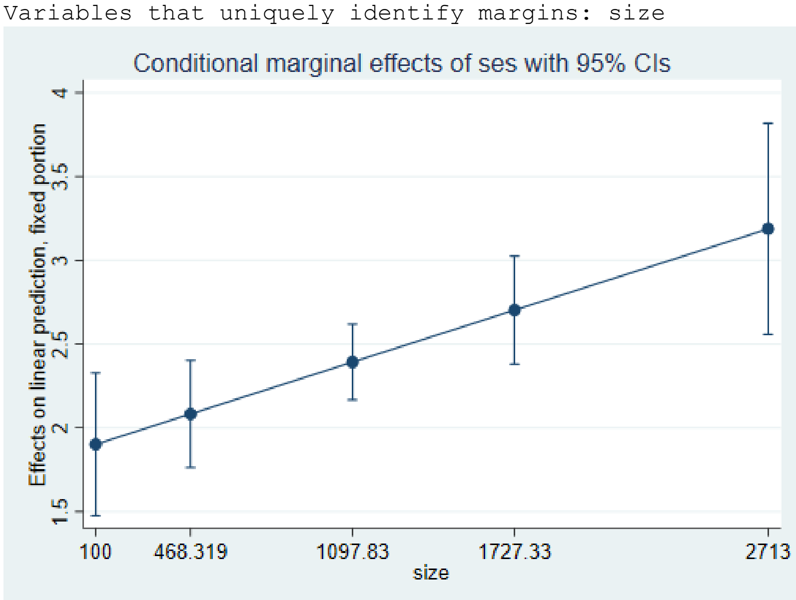
```
. di e(b)[1,1]-e(b)[1,3]*$sizesd
2.0952691
```

Since SES is continuous, we may want to show these differences graphically. For predicted values and graphs, it is better to use uncentered size, even though we would present the coefficients from the centered model.

```
. mixed mathach c.ses##c.size || id: ses, cov(unstr)
```

```
Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(3)      =     438.94
Log likelihood =  -23314.32                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
     mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         ses |   1.852348   .2332827     7.94   0.000     1.395122    2.309573
        size |   -.000266   .0003041    -0.87   0.382    -.0008621    .0003301
             |
c.ses#c.size |   .0004921   .0001853     2.65   0.008     .0001288    .0008554
             |
       _cons |   12.96714   .3817026    33.97   0.000     12.21902    13.71526
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    var(ses) |   .3168798    .221958      .0802919    1.250597
                  var(_cons) |   4.788075   .6640414       3.64847    6.283637
              cov(ses,_cons) |  -.1128863   .2887151     -.6787576    .4529849
-----------------------------+------------------------------------------------
               var(Residual) |   36.82078    .628957      35.60845    38.07438
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 461.36                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. sum size if tagged==1

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        size |        160    1097.825    629.5064        100       2713

. global sizemin=r(min)

. global sizemax=r(max)

. global sizemean=r(mean)

. global sizeplussd=r(mean)+r(sd)

. global sizeminussd=r(mean)-r(sd)

. margins, dydx(ses) at(size=($sizemin $sizeminussd $sizemean $sizeplussd $sizemax))
atmeans

Conditional marginal effects                              Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  ses
1._at: ses  = .0001434 (mean)
       size =      100
2._at: ses  = .0001434 (mean)
       size = 468.3186
3._at: ses  = .0001434 (mean)
       size = 1097.825
4._at: ses  = .0001434 (mean)
```

```
         size = 1727.331
5._at: ses  = .0001434 (mean)
         size =      2713

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ses          |
         _at |
          1  |   1.901557   .2173656     8.75   0.000     1.475528    2.327585
          2  |   2.082803   .1635671    12.73   0.000     1.762217    2.403388
          3  |   2.392577   .1153567    20.74   0.000     2.166482    2.618672
          4  |   2.702351   .1645847    16.42   0.000     2.379771    3.024931
          5  |   3.187389   .3214957     9.91   0.000     2.557269    3.817509
------------------------------------------------------------------------------

. marginsplot

Variables that uniquely identify margins: size
```

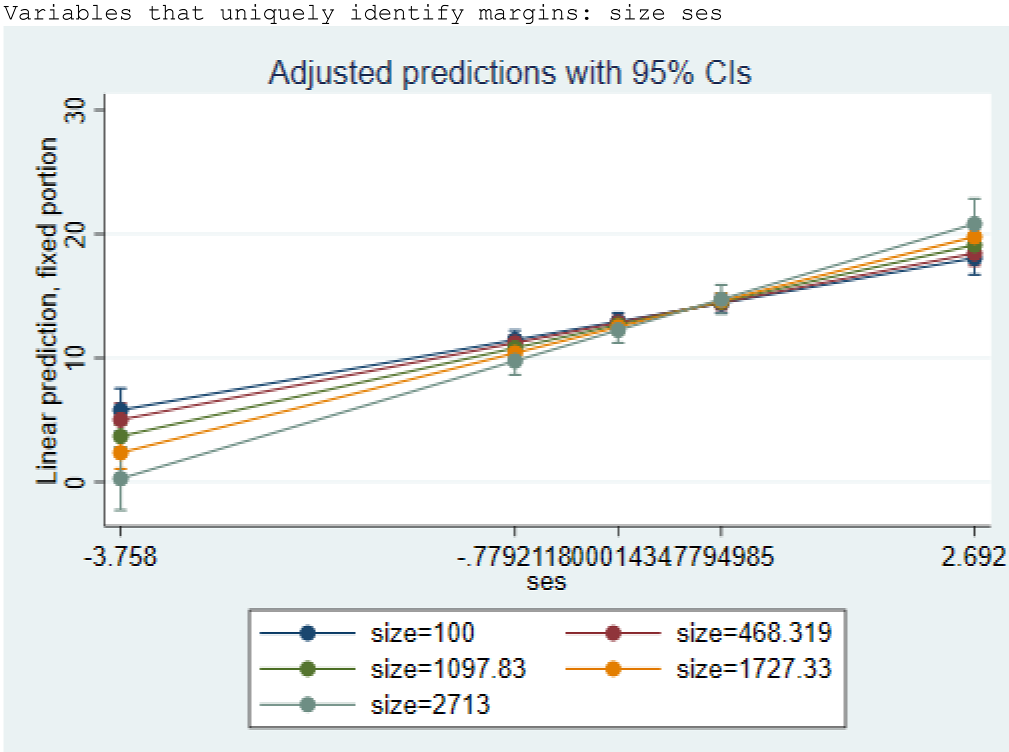


This shows the entire range of effect sizes for SES depending on size. We can see those as slopes if we calculate predicted values and plot them, but for that, let's also save the minimum, maximum etc. for SES:

```
. sum ses

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
         ses |      7,185    .0001434    .7793552     -3.758      2.692

. global sesmin=r(min)

. global sesmax=r(max)

. global sesmean=r(mean)

. global sesplussd=r(mean)+r(sd)

. global sesminussd=r(mean)-r(sd)
```

```
. margins, at(size=($sizemin $sizeminussd $sizemean $sizeplussd $sizemax) ses=($sesmin
$sesminussd $sesmean $sesplussd $sesmax)) atmeans

Adjusted predictions                                    Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
1._at:  ses  =    -3.758
        size =       100
2._at:  ses  =    -3.758
        size =  468.3186
3._at:  ses  =    -3.758
        size =  1097.825
4._at:  ses  =    -3.758
        size =  1727.331
5._at:  ses  =    -3.758
        size =      2713
6._at:  ses  = -.7792118
        size =       100
7._at:  ses  = -.7792118
        size =  468.3186
8._at:  ses  = -.7792118
        size =  1097.825
9._at:  ses  = -.7792118
        size =  1727.331
10._at: ses  = -.7792118
        size =      2713
11._at: ses  =  .0001434
        size =       100
12._at: ses  =  .0001434
        size =  468.3186
13._at: ses  =  .0001434
        size =  1097.825
14._at: ses  =  .0001434
        size =  1727.331
15._at: ses  =  .0001434
        size =      2713
16._at: ses  =  .7794985
        size =       100
17._at: ses  =  .7794985
        size =  468.3186
18._at: ses  =  .7794985
        size =  1097.825
19._at: ses  =  .7794985
        size =  1727.331
20._at: ses  =  .7794985
        size =      2713
21._at: ses  =     2.692
        size =       100
22._at: ses  =     2.692
        size =  468.3186
23._at: ses  =     2.692
        size =  1097.825
24._at: ses  =     2.692
        size =  1727.331
25._at: ses  =     2.692
        size =      2713

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   5.794493   .8937958     6.48   0.000     4.042685      7.5463
```

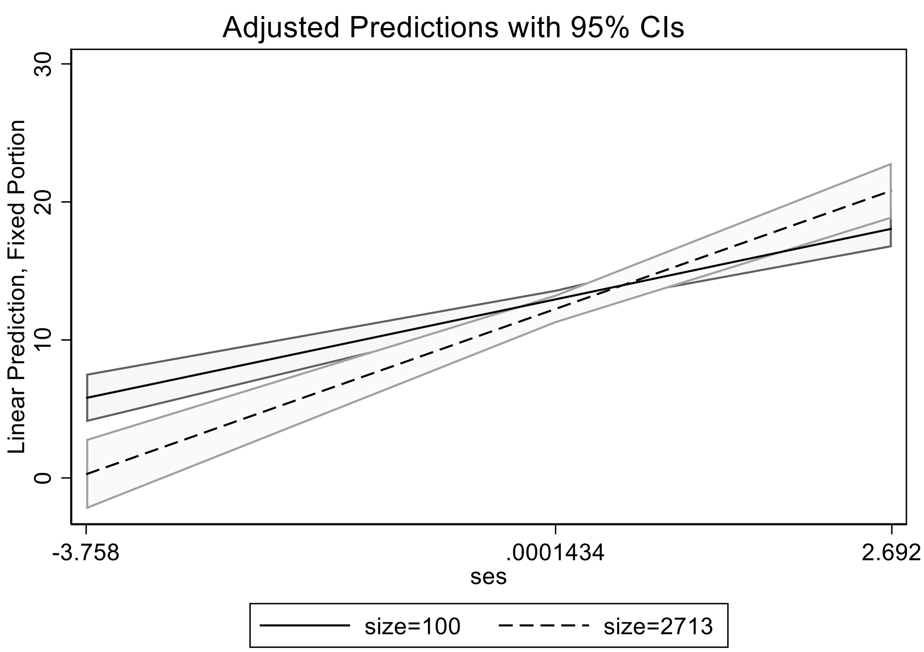
```
        2  |    5.01541    .676854     7.41   0.000       3.6888    6.342019
        3  |   3.683851   .4786823     7.70   0.000     2.745651    4.622051
        4  |   2.352292   .6678443     3.52   0.000     1.043341    3.661243
        5  |   .2673639   1.296626     0.21   0.837    -2.273976    2.808704
        6  |   11.45883    .395233    28.99   0.000     10.68418    12.23347
        7  |   11.21964   .2993116    37.48   0.000       10.633    11.80628
        8  |   10.81083   .2120376    50.99   0.000     10.39524    11.22642
        9  |   10.40202   .2961318    35.13   0.000     9.821614    10.98243
       10  |   9.761919   .5745359    16.99   0.000      8.63585    10.88799
       11  |   12.94081   .3556007    36.39   0.000     12.24385    13.63778
       12  |   12.84288   .2674408    48.02   0.000     12.31871    13.36705
       13  |    12.6755   .1890731    67.04   0.000     12.30492    13.04607
       14  |   12.50811   .2707095    46.20   0.000     11.97753    13.03869
       15  |   12.24603   .5285042    23.17   0.000     11.21018    13.28188
       16  |    14.4228   .3925439    36.74   0.000     13.65343    15.19217
       17  |   14.46612   .2931934    49.34   0.000     13.89147    15.04077
       18  |   14.54016   .2066463    70.36   0.000     14.13514    14.94518
       19  |    14.6142   .3029515    48.24   0.000     14.02043    15.20798
       20  |   14.73013   .5950645    24.75   0.000     13.56383    15.89644
       21  |   18.05953   .6820231    26.48   0.000     16.72279    19.39627
       22  |   18.44949   .5090456    36.24   0.000     17.45177     19.4472
       23  |   19.11597   .3581556    53.37   0.000       18.414    19.81794
       24  |   19.78245    .525949    37.61   0.000     18.75161    20.81329
       25  |   20.82602    1.03428    20.14   0.000     18.79887    22.85317
------------------------------------------------------------------------------

. marginsplot, x(ses)

Variables that uniquely identify margins: size ses
```



This is a bit too busy, let’s simplify to look at two extremes, minimum size and maximum size:

```
. margins, at(size=($sizemin $sizemax) ses=($sesmin $sesmean $sesmax)) atmeans

Adjusted predictions                            Number of obs     =      7,185
```

```
Expression   : Linear prediction, fixed portion, predict()
1._at        : ses             =      -3.758
               size            =         100

2._at        : ses             =      -3.758
               size            =        2713

3._at        : ses             =    .0001434
               size            =         100

4._at        : ses             =    .0001434
               size            =        2713

5._at        : ses             =       2.692
               size            =         100

6._at        : ses             =       2.692
               size            =        2713

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   5.794493   .8937958     6.48   0.000     4.042685      7.5463
          2  |   .2673639   1.296626     0.21   0.837    -2.273976    2.808704
          3  |   12.94081   .3556007    36.39   0.000     12.24385    13.63778
          4  |   12.24603   .5285042    23.17   0.000     11.21018    13.28188
          5  |   18.05953   .6820231    26.48   0.000     16.72279    19.39627
          6  |   20.82602    1.03428    20.14   0.000     18.79887    22.85317
------------------------------------------------------------------------------

. marginsplot, x(ses) plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5))
scheme(s1mono)

  Variables that uniquely identify margins: size ses
```



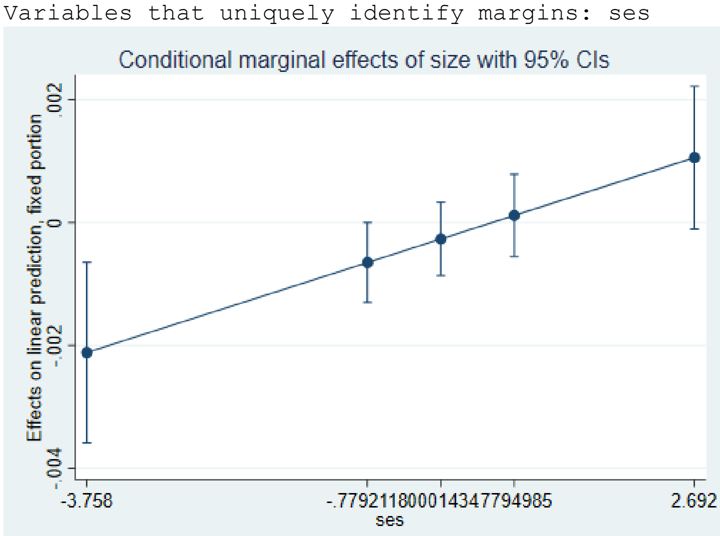
SIZE as the focal variable, SES as the moderator:

For a student with mean SES, when school size increases by 1 student, math achievement does not change – the main effect of school size is -.000266 but it's not statistically significant. Let's use margins to see what that effect of size looks like at other values of SES:

```
. margins, dydx(size) at(ses=($sesmin $sesminussd $sesmean $sesplussd $sesmax))
atmeans

Conditional marginal effects                            Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
dy/dx wrt:  size
1._at: ses  =    -3.758
       size =  1056.862 (mean)
2._at: ses  = -.7792118
       size =  1056.862 (mean)
3._at: ses  =  .0001434
       size =  1056.862 (mean)
4._at: ses  =  .7794985
       size =  1056.862 (mean)
5._at: ses  =     2.692
       size =  1056.862 (mean)
------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
size         |
         _at |
          1  |  -.0021152   .0007501    -2.82   0.005    -.0035853   -.0006452
          2  |  -.0006494    .000332    -1.96   0.050    -.0013001    1.31e-06
          3  |  -.0002659   .0003041    -0.87   0.382     -.000862    .0003302
          4  |   .0001176   .0003413     0.34   0.730    -.0005514    .0007866
          5  |   .0010587   .0005935     1.78   0.074    -.0001046     .002222
------------------------------------------------------------------------------
```

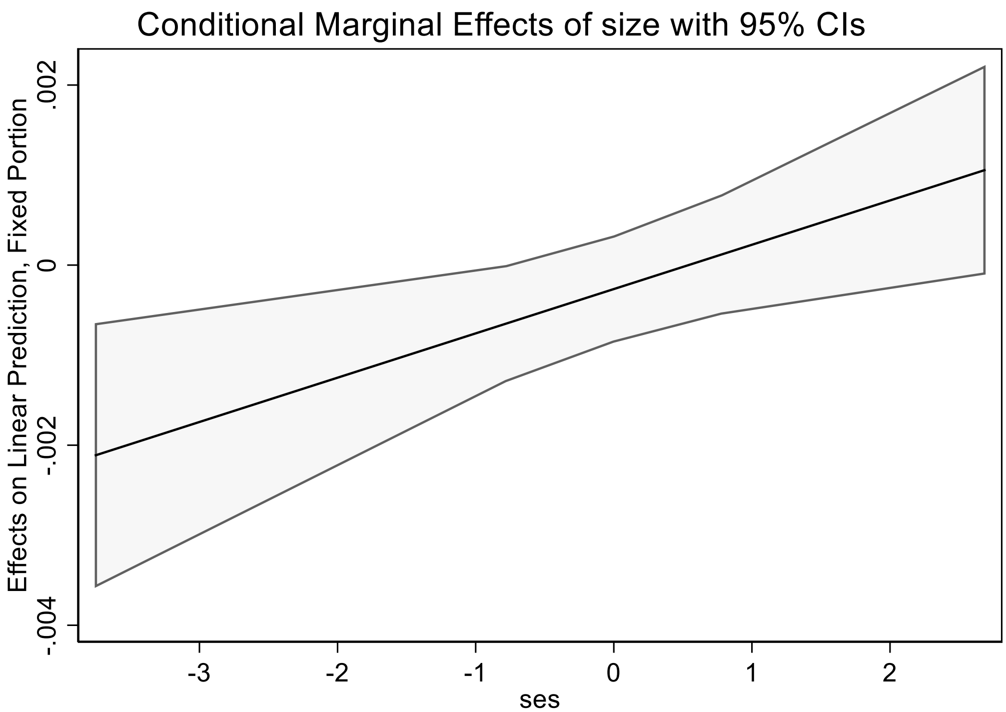
Here we can see that the effect of school size is only significant for those with SES lower than 1 SD below the mean (the p value is exactly .05 for 1 SD below the mean, so we assume it will be significant anywhere below that). Let's see that difference in effect sizes graphically:

```
. marginsplot

Variables that uniquely identify margins: ses
```



We can see which effects are statistically significant based on whether 0 is within the confidence interval or not. That might be better visible with a confidence band:

```
. marginsplot, x(ses) plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5))
>  scheme(s1mono) xlabel(-3 -2 -1 0 1 2)

  Variables that uniquely identify margins: ses
```

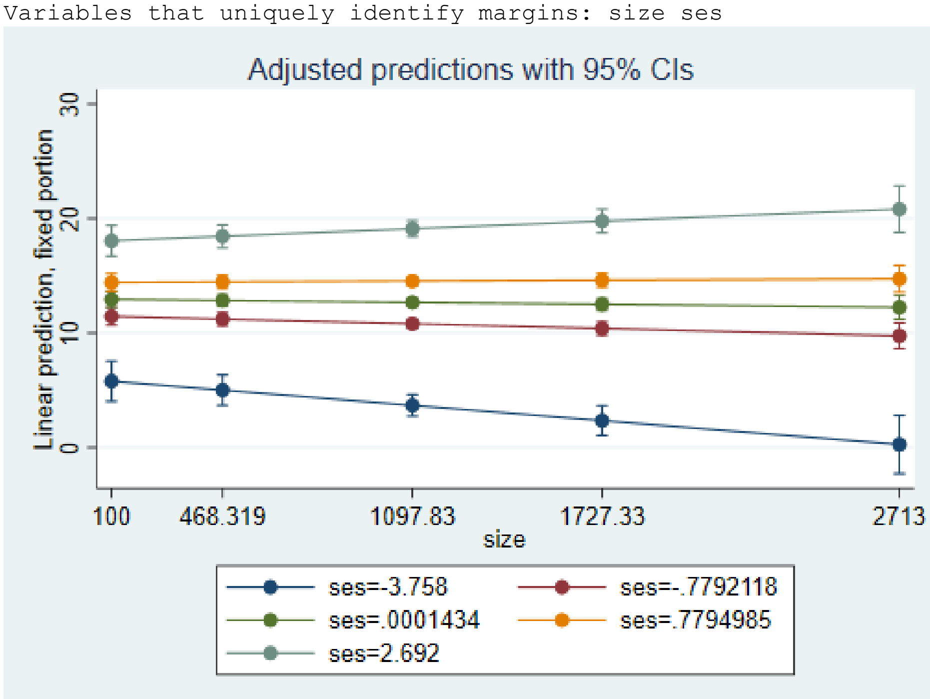


Finally, let's see those effects of size as actual slopes:

```
. margins, at(size=($sizemin $sizeminussd $sizemean $sizeplussd $sizemax) ses=($sesmin
$sesminussd $sesmean $sesplussd $sesmax)) atmeans

Adjusted predictions                                   Number of obs = 7,185

Expression: Linear prediction, fixed portion, predict()
1._at:  ses  =    -3.758
        size =       100
2._at:  ses  =    -3.758
        size =  468.3186
3._at:  ses  =    -3.758
        size =  1097.825
4._at:  ses  =    -3.758
        size =  1727.331
5._at:  ses  =    -3.758
        size =      2713
6._at:  ses  = -.7792118
        size =       100
7._at:  ses  = -.7792118
        size =  468.3186
8._at:  ses  = -.7792118
        size =  1097.825
9._at:  ses  = -.7792118
```

```
        size =  1727.331
10._at: ses  = -.7792118
        size =      2713
11._at: ses  =  .0001434
        size =       100
12._at: ses  =  .0001434
        size =  468.3186
13._at: ses  =  .0001434
        size =  1097.825
14._at: ses  =  .0001434
        size =  1727.331
15._at: ses  =  .0001434
        size =      2713
16._at: ses  =  .7794985
        size =       100
17._at: ses  =  .7794985
        size =  468.3186
18._at: ses  =  .7794985
        size =  1097.825
19._at: ses  =  .7794985
        size =  1727.331
20._at: ses  =  .7794985
        size =      2713
21._at: ses  =     2.692
        size =       100
22._at: ses  =     2.692
        size =  468.3186
23._at: ses  =     2.692
        size =  1097.825
24._at: ses  =     2.692
        size =  1727.331
25._at: ses  =     2.692
        size =      2713

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   5.794493   .8937958     6.48   0.000     4.042685      7.5463
          2  |    5.01541    .676854     7.41   0.000       3.6888    6.342019
          3  |   3.683851   .4786823     7.70   0.000     2.745651    4.622051
          4  |   2.352292   .6678443     3.52   0.000     1.043341    3.661243
          5  |   .2673639   1.296626     0.21   0.837    -2.273976    2.808704
          6  |   11.45883    .395233    28.99   0.000     10.68418    12.23347
          7  |   11.21964   .2993116    37.48   0.000       10.633    11.80628
          8  |   10.81083   .2120376    50.99   0.000     10.39524    11.22642
          9  |   10.40202   .2961318    35.13   0.000     9.821614    10.98243
         10  |   9.761919   .5745359    16.99   0.000      8.63585    10.88799
         11  |   12.94081   .3556007    36.39   0.000     12.24385    13.63778
         12  |   12.84288   .2674408    48.02   0.000     12.31871    13.36705
         13  |    12.6755   .1890731    67.04   0.000     12.30492    13.04607
         14  |   12.50811   .2707095    46.20   0.000     11.97753    13.03869
         15  |   12.24603   .5285042    23.17   0.000     11.21018    13.28188
         16  |    14.4228   .3925439    36.74   0.000     13.65343    15.19217
         17  |   14.46612   .2931934    49.34   0.000     13.89147    15.04077
         18  |   14.54016   .2066463    70.36   0.000     14.13514    14.94518
         19  |    14.6142   .3029515    48.24   0.000     14.02043    15.20798
         20  |   14.73013   .5950645    24.75   0.000     13.56383    15.89644
         21  |   18.05953   .6820231    26.48   0.000     16.72279    19.39627
         22  |   18.44949   .5090456    36.24   0.000     17.45177     19.4472
         23  |   19.11597   .3581556    53.37   0.000       18.414    19.81794
         24  |   19.78245    .525949    37.61   0.000     18.75161    20.81329
```

```
         25  |   20.82602    1.03428    20.14   0.000     18.79887    22.85317
------------------------------------------------------------------------------

. marginsplot, x(size)

Variables that uniquely identify margins: size ses
```




```
. marginsplot, x(size) plotop(msymbol(i)) recastci(rarea) ciopts(fintensity(5))
scheme(s1mono)

Variables that uniquely identify margins: size ses
```