SOCY7708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: HLM Model Building Strategies: Example

Let's follow the second (combined) model building strategy and evaluate a model where we consider two level 1 predictors – ses and female, and two level 2 predictors – sector and a set of dummies for school size. (Note that we could also consider two additional level 2 predictors that are aggregates of level 1 predictors – average school-level SES and gender composition of school; but we keep it simple for now.)

```
. mixed mathach c.ses##c.sector c.ses##i.sized  i.female##i.sector i.female##i.sized
|| id: ses female, cov(unstr)
note: ses omitted because of collinearity.
note: 1.sector omitted because of collinearity.
Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     677.31
Log likelihood = -23246.527                     Prob > chi2       =     0.0000
----------------------------------------------------------------------------------
         mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+----------------------------------------------------------------
             ses |   3.071058   .2823452    10.88   0.000     2.517671    3.624444
          sector |    2.36977   .4358575     5.44   0.000     1.515505    3.224035
                 |
c.ses#c.sector |  -1.283094   .2364085    -5.43   0.000    -1.746446   -.8197415
                 |
             ses |          0  (omitted)
                 |
           sized |
              2  |    1.23097   .5676327     2.17   0.030     .1184298    2.343509
              3  |   1.752608    .585621     2.99   0.003     .6048122    2.900404
                 |
     sized#c.ses |
              2  |  -.2519453   .2923376    -0.86   0.389    -.8249165    .3210258
              3  |  -.1277427   .3097203    -0.41   0.680    -.7347833    .4792979
                 |
        1.female |  -.2816415   .4703985    -0.60   0.549    -1.203606    .6403227
        1.sector |          0  (omitted)
                 |
   female#sector |
            1 1  |  -.1739571   .4058262    -0.43   0.668    -.9693619    .6214477
                 |
    female#sized |
            1 2  |   -.712411   .5186187    -1.37   0.170    -1.728885    .3040631
            1 3  |  -1.308136   .5277861    -2.48   0.013    -2.342578   -.2736945
                 |
           _cons |   11.03738   .5302974    20.81   0.000     9.998014    12.07674
----------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(ses_m)  |   .0864968          .             .           .
                var(female)  |   .7588742          .             .           .
                 var(_cons)  |   4.090261          .             .           .
          cov(ses_m,female)  |  -.1480118          .             .           .
```

```
          cov(ses_m,_cons) |   .5930792          .           .           .
         cov(female,_cons) |  -.9053627          .           .           .
-----------------------------+------------------------------------------------
             var(Residual) |   36.36132          .           .           .
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 311.08                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
Warning: Standard-error calculation failed.
```

This model has problems with variance components – let's test each of them. First ses:

```
. mixed mathach c.ses##c.sector c.ses##i.sized  i.female##i.sector i.female##i.sized
|| id: ses, cov(unstr)
note: ses omitted because of collinearity.
note: 1.sector omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     703.22
Log likelihood = -23248.671                     Prob > chi2       =     0.0000
--------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+----------------------------------------------------------------
           ses |   3.048262   .2824976    10.79   0.000     2.494577    3.601947
        sector |   2.370088   .4077368     5.81   0.000     1.570938    3.169237
               |
c.ses#c.sector |  -1.271897   .2364572    -5.38   0.000    -1.735345   -.8084498
               |
           ses |          0  (omitted)
               |
         sized |
            2  |   1.214981   .5279932     2.30   0.021     .1801335    2.249829
            3  |   1.718568   .5468995     3.14   0.002      .646665    2.790472
               |
   sized#c.ses |
            2  |  -.2372528   .2926702    -0.81   0.418    -.8108759    .3363703
            3  |  -.1103129   .3099481    -0.36   0.722       -.7178    .4971742
               |
      1.female |  -.2954447   .4224582    -0.70   0.484    -1.123448    .5325581
      1.sector |          0  (omitted)
               |
 female#sector |
          1 1  |  -.1500386    .368775    -0.41   0.684    -.8728244    .5727472
               |
  female#sized |
          1 2  |  -.6956448   .4670362    -1.49   0.136    -1.611019    .2197294
          1 3  |  -1.285465   .4759068    -2.70   0.007    -2.218225   -.3527048
               |
         _cons |   11.05523   .4953209    22.32   0.000     10.08442    12.02604
--------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    var(ses) |   .0843152   .0868693      .0111923    .6351725
                  var(_cons) |   3.467264   .5180921      2.587004    4.647044
             cov(ses,_cons) |   .5406851   .2936755     -.0349084    1.116279
-----------------------------+------------------------------------------------
```

```
              var(Residual) |   36.48905   .6177096      35.29823    37.72005
-----------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 306.79            Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
           . |      7,185          .  -23248.67      16   46529.34   46639.42
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. est store sesrandom

. mixed mathach c.ses##c.sector c.ses##i.sized  i.female##i.sector i.female##i.sized
|| id:
note: ses omitted because of collinearity.
note: 1.sector omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     699.99
Log likelihood = -23251.547                     Prob > chi2       =     0.0000

-------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+---------------------------------------------------------------
           ses |   3.049839   .2763021    11.04   0.000     2.508297    3.591381
        sector |   2.371117   .4019846     5.90   0.000     1.583241    3.158992
               |
c.ses#c.sector |  -1.265125   .2317514    -5.46   0.000     -1.71935    -.810901
               |
           ses |          0  (omitted)
               |
         sized |
            2  |   1.237825   .5204513     2.38   0.017     .2177591    2.257891
            3  |     1.7123    .538888     3.18   0.001     .6560985    2.768501
               |
   sized#c.ses |
            2  |  -.2557244   .2866414    -0.89   0.372    -.8175312    .3060824
            3  |  -.1155212    .303394    -0.38   0.703    -.7101625    .4791201
               |
      1.female |  -.2856112   .4223186    -0.68   0.499     -1.11334    .5421179
      1.sector |          0  (omitted)
               |
 female#sector |
          1 1  |  -.1414109   .3692235    -0.38   0.702    -.8650756    .5822539
               |
  female#sized |
          1 2  |  -.7326045   .4664619    -1.57   0.116    -1.646853    .1816441
          1 3  |  -1.283305   .4760334    -2.70   0.007    -2.216314   -.3502968
               |
         _cons |    11.0998   .4879931    22.75   0.000     10.14335    12.05625
-------------------------------------------------------------------------------
```

```
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   3.317526   .4858015      2.489824    4.420383
-----------------------------+------------------------------------------------
               var(Residual) |   36.55247   .6172766      35.36243    37.78255
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 301.04        Prob >= chibar2 = 0.0000

. est store noslope

. lrtest sesrandom noslope

Likelihood-ratio test
Assumption: noslope nested within sesrandom

 LR chi2(2) =   5.75
Prob > chi2 = 0.0564

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space. If this is not true, then the reported test is
conservative.

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
           . |      7,185          .  -23251.55      14   46531.09   46627.41
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.
```

BIC value is 12 points lower in the model without SES variance so it's preferred based on both BIC and LR test. Now let's test the slope variance for female:

```
. mixed mathach c.ses##c.sector c.ses##i.sized  i.female##i.sector i.female##i.sized
|| id: female, cov(unstr)
note: ses omitted because of collinearity.
note: 1.sector omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     696.49
Log likelihood = -23249.266                     Prob > chi2       =     0.0000
--------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+----------------------------------------------------------------
           ses |   3.072735   .2756691    11.15   0.000     2.532434    3.613037
        sector |   2.367268    .432287     5.48   0.000     1.520001    3.214535
               |
c.ses#c.sector |  -1.276275   .2314061    -5.52   0.000    -1.729823   -.8227277
               |
           ses |          0  (omitted)
               |
```

```
       sized |
          2  |   1.276582   .5631707     2.27   0.023     .1727874    2.380376
          3  |   1.766474   .5807221     3.04   0.002     .6282794    2.904668
             |
 sized#c.ses |
          2  |  -.2675328     .28581    -0.94   0.349    -.8277101    .2926445
          3  |  -.1308607   .3026749    -0.43   0.665    -.7240926    .4623711
             |
    1.female |  -.2465819   .4707665    -0.52   0.600    -1.169267    .6761034
    1.sector |          0  (omitted)
             |
female#sector |
        1 1  |  -.1725857   .4063304    -0.42   0.671    -.9689787    .6238073
             |
 female#sized |
        1 2  |  -.7773901   .5187856    -1.50   0.134    -1.794191    .2394109
        1 3  |  -1.331266   .5283803    -2.52   0.012    -2.366872   -.2956592
             |
       _cons |   11.05971   .5258146    21.03   0.000     10.02914    12.09029
--------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                 var(female) |    .768301   .5565779      .1857369    3.178079
                  var(_cons) |   3.992242   .6855878      2.851281    5.589767
          cov(female,_cons) |   -.959724   .5169883     -1.973003    .0535545
-----------------------------+------------------------------------------------
               var(Residual) |   36.43078   .6198063      35.23601    37.66606
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 305.60                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. est store femalerandom

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
femalerandom |      7,185          .  -23249.27      16   46530.53   46640.61
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. lrtest noslope femalerandom

Likelihood-ratio test
Assumption: noslope nested within femalerandom

 LR chi2(2) =   4.56
Prob > chi2 = 0.1022

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space. If this is not true, then the reported test is
conservative.
```

BIC value is 13 points lower in the model without FEMALE variance so it's preferred based on both BIC and LR test. We conclude that neither level 1 predictor needs a random slope. However, we still want to investigate the cross-level interactions, so we estimate:

```
. mixed mathach c.ses##c.sector c.ses##i.sized i.female##i.sector i.female##i.sized ||
id:
note: ses omitted because of collinearity.
note: 1.sector omitted because of collinearity.

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(11)     =     699.99
Log likelihood = -23251.547                     Prob > chi2       =     0.0000
--------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+----------------------------------------------------------------
           ses |   3.049839   .2763021    11.04   0.000     2.508297    3.591381
        sector |   2.371117   .4019846     5.90   0.000     1.583241    3.158992
               |
c.ses#c.sector |  -1.265125   .2317514    -5.46   0.000     -1.71935    -.810901
               |
           ses |          0  (omitted)
               |
         sized |
            2  |   1.237825   .5204513     2.38   0.017     .2177591    2.257891
            3  |     1.7123    .538888     3.18   0.001     .6560985    2.768501
               |
   sized#c.ses |
            2  |  -.2557244   .2866414    -0.89   0.372    -.8175312    .3060824
            3  |  -.1155212    .303394    -0.38   0.703    -.7101625    .4791201
               |
      1.female |  -.2856112   .4223186    -0.68   0.499     -1.11334    .5421179
      1.sector |          0  (omitted)
               |
 female#sector |
          1 1  |  -.1414109   .3692235    -0.38   0.702    -.8650756    .5822539
               |
  female#sized |
          1 2  |  -.7326045   .4664619    -1.57   0.116    -1.646853    .1816441
          1 3  |  -1.283305   .4760334    -2.70   0.007    -2.216314   -.3502968
               |
         _cons |    11.0998   .4879931    22.75   0.000     10.14335    12.05625
--------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   3.317526   .4858015      2.489824    4.420383
-----------------------------+------------------------------------------------
               var(Residual) |   36.55247   .6172766      35.36243    37.78255
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 301.04        Prob >= chibar2 = 0.0000
```

Female main effect is not significant, but because there are significant interactions with it, we'd keep it. Let's test for joint significance of other non-significant terms (doesn't include non-significant dummies in sets of dummies, sets should be evaluated as a whole):

```
. test 1.female#1.sector=0

 ( 1)  [mathach]1.female#1.sector = 0
```

```
          chi2(  1) =    0.15
        Prob > chi2 =    0.7017

. test 2.sized#c.ses=0, acc

 ( 1)  [mathach]1.female#1.sector = 0
 ( 2)  [mathach]2.sized#c.ses = 0

          chi2(  2) =    0.94
        Prob > chi2 =    0.6240

. test 3.sized#c.ses=0, acc

 ( 1)  [mathach]1.female#1.sector = 0
 ( 2)  [mathach]2.sized#c.ses = 0
 ( 3)  [mathach]3.sized#c.ses = 0

          chi2(  3) =    1.01
        Prob > chi2 =    0.7990
```

We can jointly omit all those:

```
. mixed mathach c.ses##c.sector i.female##i.sized || id:

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(8)      =     698.49
Log likelihood = -23252.051                     Prob > chi2       =     0.0000
--------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+----------------------------------------------------------------
           ses |   2.906627   .1400363    20.76   0.000     2.632161    3.181093
        sector |   2.292641   .3582754     6.40   0.000     1.590434    2.994848
               |
c.ses#c.sector |   -1.27794   .2109064    -6.06   0.000    -1.691309   -.8645712
               |
      1.female |  -.3719948    .385254    -0.97   0.334    -1.127079    .3830891
               |
         sized |
            2  |   1.234907   .5191091     2.38   0.017      .217472    2.252342
            3  |   1.692167   .5386236     3.14   0.002     .6364839     2.74785
               |
  female#sized |
          1 2  |  -.7090223   .4646167    -1.53   0.127    -1.619654    .2016097
          1 3  |  -1.219811   .4604496    -2.65   0.008    -2.122276   -.3173464
               |
         _cons |   11.13458   .4845513    22.98   0.000     10.18488    12.08428
--------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   3.326955   .4861819      2.498373    4.430335
-----------------------------+------------------------------------------------
               var(Residual) |   36.55583   .6173111      35.36572    37.78598
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 302.48        Prob >= chibar2 = 0.0000
```

Finally, we should consider whether we want to simplify those coding of size – could we combine groups 2 and 3?

```
. test 2.sized=3.sized

 ( 1)  [mathach]2.sized - [mathach]3.sized = 0

           chi2(  1) =    1.14
         Prob > chi2 =    0.2865

. test 1.female#2.sized=1.female#3.sized, acc

 ( 1)  [mathach]2.sized - [mathach]3.sized = 0
 ( 2)  [mathach]1.female#2.sized - [mathach]1.female#3.sized = 0

           chi2(  2) =    2.27
         Prob > chi2 =    0.3212
```

Indeed, they don' produce anything distinct, so we can combine them and use a single dummy for size – small vs large.

```
. tab sized, gen(sized_)

  RECODE of |
       size |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      1,524       21.21       21.21
          2 |      2,984       41.53       62.74
          3 |      2,677       37.26      100.00
------------+-----------------------------------
      Total |      7,185      100.00

. tab sized_1

    sized== |
     1.0000 |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      5,661       78.79       78.79
          1 |      1,524       21.21      100.00
------------+-----------------------------------
      Total |      7,185      100.00
```

It is coded 1=small, 0=large. We could also reverse code it if we'd like. So here is our final parsimonious model. I will also compare it to the original one:

```
. mixed mathach c.ses##c.sector i.female##i.sized_1 || id:

Mixed-effects ML regression                     Number of obs     =      7,185
Group variable: id                              Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
                                                Wald chi2(6)      =     696.34
Log likelihood = -23253.187                     Prob > chi2       =     0.0000
--------------------------------------------------------------------------------
       mathach | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+----------------------------------------------------------------
           ses |   2.908528   .1400094    20.77   0.000     2.634115    3.182942
        sector |   2.227118   .3332109     6.68   0.000     1.574037      2.8802
               |
```

```
c.ses#c.sector |  -1.284285   .2107547    -6.09   0.000    -1.697357   -.8712136
               |
      1.female |  -1.345095   .1811197    -7.43   0.000    -1.700083    -.990107
     1.sized_1 |  -1.451899   .4824572    -3.01   0.003    -2.397498   -.5063004
               |
female#sized_1 |
          1 1  |   .9753494   .4256332     2.29   0.022     .1411236    1.809575
               |
         _cons |   12.62697   .2444691    51.65   0.000     12.14782    13.10612
--------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   3.320192   .4849929      2.493589    4.420805
-----------------------------+------------------------------------------------
               var(Residual) |   36.56893   .6175188      35.37842    37.79949
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 303.16        Prob >= chibar2 = 0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
           . |      7,185          .  -23253.19       9   46524.37   46586.29
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. lrtest . femalerandom

Likelihood-ratio test
Assumption: . nested within femalerandom

 LR chi2(7) =   7.84
Prob > chi2 = 0.3467

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space. If this is not true, then the reported test is
conservative.

. lrtest . sesrandom

Likelihood-ratio test
Assumption: . nested within sesrandom

 LR chi2(7) =   9.03
Prob > chi2 = 0.2504

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space. If this is not true, then the reported test is
conservative.
```

BIC is more than 40 points lower for this model than for the initial models with SES or FEMALE random slope, and LR test confirms that this parsimonious model fits just as well as those more complex ones, so we stick to this one.