SOCY7708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: Three Level Models

A 3-level HLM model is a logical extension of the 2-level model. Similar model building strategies apply, although now both level 1 and level 2 slopes may vary.

The data for our example are stored in eg1.dta, eg2.dta, and eg3.dta. These data consist of 7230 observations collected on 1721 children from 60 schools, beginning at the end of grade one and followed up annually thereafter until grade six. The students are the level-2 units, the schools are the level-3 units. The outcome variable of interest is the result of a math test, represented by the variable MATH. On level 1, the information available includes the year of the study (denoted by the variable YEAR), the grade level at each testing occasion (denoted by GRADE) and an indicator that a child is retained in grade for a particular year (denoted by the variable RETAINED, which assumes a value of 1 if retained, 0 otherwise). At level-2, the following variables are available: GENDER, denoting the gender of the child (1=female), BLACK, denoting whether the child is Black, and HISPANIC, denoting whether the child is Hispanic. At level-3, the following variables are available: SIZE, measuring the size of school, LOWINC indicating the percentage of low-income students in the school, and MOBILITY indicating the level of residential mobility on the school level (%).

Let’s merge the three files; we will use variables CHILDID and SCHOOLID to match across levels.

```
. merge m:1 schoolid childid using eg2.dta

    Result                           # of obs.
    -----------------------------------------
    not matched                             0
    matched                             7,230  (_merge==3)
    -----------------------------------------

. drop _merge

. merge m:1 schoolid using eg3.dta

    Result                           # of obs.
    -----------------------------------------
    not matched                             0
    matched                             7,230  (_merge==3)
    -----------------------------------------

. drop _merge
```

Let’s start by modeling the linear growth trajectory unconditionally and without random effects at levels 2 and 3:

```
. tab year

       year |      Freq.     Percent        Cum.
------------+-----------------------------------
       1.00 |        131        1.81        1.81
```

```
       2.00 |      1,346       18.62       20.43
       3.00 |      1,520       21.02       41.45
       4.00 |      1,672       23.13       64.58
       5.00 |      1,387       19.18       83.76
       6.00 |      1,174       16.24      100.00
------------+-----------------------------------
      Total |      7,230      100.00

. gen year0=year-1

. mixed math year0 || schoolid: || childid:

Mixed-effects ML regression                     Number of obs     =      7,230
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
       schoolid |         60         18      120.5        387
        childid |      1,721          2        4.2          6
-------------------------------------------------------------

                                                Wald chi2(1)      =   19120.98
Log likelihood = -8373.5215                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |   .7461301   .0053958   138.28   0.000     .7355545    .7567058
       _cons |  -2.645932   .0624743   -42.35   0.000     -2.76838   -2.523485
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolid: Identity           |
                  var(_cons) |   .1832551   .0396317      .1199419    .2799893
-----------------------------+------------------------------------------------
childid: Identity            |
                  var(_cons) |    .669919   .0262842      .6203339    .7234676
-----------------------------+------------------------------------------------
               var(Residual) |   .3469397     .00661      .3342232    .3601401
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 5174.77               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat icc

Residual intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                    schoolid |   .1526981   .0282748      .1050886    .2166549
            childid|schoolid |    .710911   .0121012      .6866291    .7340402
------------------------------------------------------------------------------

. est store basic3

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
```

```
-------------+---------------------------------------------------------------
      basic3 |      7,230          .  -8373.522       5   16757.04   16791.47
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. mixed math year0 || schoolid:

Mixed-effects ML regression                     Number of obs     =      7,230
Group variable: schoolid                        Number of groups  =         60

                                                Obs per group:
                                                              min =         18
                                                              avg =      120.5
                                                              max =        387

                                                Wald chi2(1)      =    7756.87
Log likelihood = -10343.209                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |    .751992   .0085383    88.07   0.000     .7352573    .7687267
       _cons |  -2.649882   .0652758   -40.60   0.000     -2.77782   -2.521944
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolid: Identity           |
                  var(_cons) |    .207243   .0401772      .1417298    .3030389
-----------------------------+------------------------------------------------
               var(Residual) |   .9978508   .0166657      .9657154    1.031056
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 1235.39       Prob >= chibar2 = 0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
           . |      7,230          .  -10343.21       4   20694.42   20721.96
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. est store basic_school

. lrtest basic3 basic_school

Likelihood-ratio test                                 LR chi2(1)  =   3939.37
(Assumption: basic_school nested in basic3)           Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space.  If this is not true, then the reported test
is conservative.

. mixed math year0 || childid:

Mixed-effects ML regression                     Number of obs     =      7,230
Group variable: childid                         Number of groups  =      1,721

                                                Obs per group:
                                                              min =          2
                                                              avg =        4.2
```

```
                                                           max =          6

                                                Wald chi2(1)      =   19156.93
Log likelihood = -8515.4376                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |   .7474524   .0054003   138.41   0.000      .736868    .7580369
       _cons |  -2.707306   .0281623   -96.13   0.000    -2.762503   -2.652109
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
childid: Identity            |
                  var(_cons) |   .8677144   .0325757      .8061596    .9339692
-----------------------------+------------------------------------------------
               var(Residual) |   .3469385   .0066102      .3342216    .3601392
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 4890.93       Prob >= chibar2 = 0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |      7,230          .  -8515.438       4    17038.88   17066.42
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. est store basic_child

. lrtest basic3 basic_child

Likelihood-ratio test                                 LR chi2(1)  =    283.83
(Assumption: basic_child nested in basic3)            Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space.  If this is not true, then the reported test
is conservative.

. mixed math year0 || schoolid: year0, cov(unstr) || childid: year0, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,230
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
       schoolid |         60         18      120.5        387
        childid |      1,721          2        4.2          6
-------------------------------------------------------------

                                                Wald chi2(1)      =    2499.85
Log likelihood = -8163.1156                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |   .7630274    .015261    50.00   0.000     .7331163    .7929384
       _cons |  -2.686874   .0568029   -47.30   0.000    -2.798205   -2.575542
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
```

```
schoolid: Unstructured       |
                 var(year0) |   .0110172   .0025621      .0069843     .017379
                 var(_cons) |    .148943   .0348396      .0941702    .2355736
           cov(year0,_cons) |  -.0104966   .0071906     -.0245899    .0035967
-----------------------------+------------------------------------------------
childid: Unstructured        |
                 var(year0) |   .0112561    .001961      .0080001    .0158373
                 var(_cons) |   .4768837   .0323937      .4174381    .5447946
           cov(year0,_cons) |   .0186451   .0062279      .0064385    .0308516
-----------------------------+------------------------------------------------
              var(Residual) |   .3014383   .0066403      .2887004    .3147382
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 5595.58              Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov

Random-effects covariance matrix for level schoolid

             |     year0      _cons
-------------+----------------------
       year0 |  .0110172
       _cons | -.0104966    .148943

Random-effects covariance matrix for level childid

             |     year0      _cons
-------------+----------------------
       year0 |  .0112561
       _cons |  .0186451   .4768837

. est store year_varies

. estat ic

Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
 year_varies |      7,230          .  -8163.116       9   16344.23   16406.21
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. lrtest basic3 year_varies

Likelihood-ratio test                                 LR chi2(4)  =    420.81
(Assumption: basic3 nested in year_varies)            Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the
boundary of the parameter space.  If this is not true, then the reported test
is conservative.
```

Let’s add level 1 variable, retained, and examine random slope for it:

```
. mixed math year0 retained || schoolid: year0 retained, cov(unstr) || childid:
year0 retained, cov(unstr)


Mixed-effects ML regression                     Number of obs     =      7,230
-------------------------------------------------------------
                |     No. of       Observations per Group
```

```
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
       schoolid |         60         18      120.5        387
        childid |      1,721          2        4.2          6
-------------------------------------------------------------

                                                Wald chi2(2)      =    2771.58
Log likelihood = -8147.2791                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |   .7625041   .0149858    50.88   0.000     .7331324    .7918758
    retained |   .0632797   .0411298     1.54   0.124    -.0173332    .1438927
       _cons |  -2.692765   .0561459   -47.96   0.000    -2.802809   -2.582721
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolid: Unstructured       |
                  var(year0) |   .0105288   .0024536      .0066684    .0166241
               var(retained) |   .0227032   .0147359      .0063621    .0810168
                  var(_cons) |   .1435571   .0340709      .0901584    .2285827
         cov(year0,retained) |  -.0089062   .0047219     -.0181609    .0003486
            cov(year0,_cons) |  -.0079595   .0069017     -.0214866    .0055675
         cov(retained,_cons) |   .0120993   .0190173     -.0251739    .0493725
-----------------------------+------------------------------------------------
childid: Unstructured        |
                  var(year0) |   .0121891   .0020074      .0088265    .0168327
               var(retained) |   .0294262   .0367675       .002542    .3406404
                  var(_cons) |   .5077876   .0338699      .4455599    .5787061
         cov(year0,retained) |   .0031353   .0072898     -.0111524     .017423
            cov(year0,_cons) |   .0145915   .0064552      .0019395    .0272436
         cov(retained,_cons) |  -.0904695   .0395096     -.1679069   -.0130321
-----------------------------+------------------------------------------------
               var(Residual) |   .2956793   .0067664      .2827105     .309243
------------------------------------------------------------------------------
LR test vs. linear model: chi2(12) = 5483.42              Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. est store retained_varies

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df        AIC        BIC
-------------+---------------------------------------------------------------
retained_v~s |      7,230          .  -8147.279      16   16326.56   16436.73
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.

. mixed math year0 retained || schoolid: year0, cov(unstr) || childid: year0,
cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,230
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
       schoolid |         60         18      120.5        387
        childid |      1,721          2        4.2          6
```

```
-------------------------------------------------------------

                                                 Wald chi2(2)      =    2519.79
Log likelihood = -8154.6735                      Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |    .764384   .0152565    50.10   0.000     .7344818    .7942863
    retained |   .1387952   .0333952     4.16   0.000     .0733417    .2042486
       _cons |  -2.698604   .0574234   -46.99   0.000    -2.811151   -2.586056
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolid: Unstructured       |
                  var(year0) |   .0110101   .0025559      .0069854    .0173538
                  var(_cons) |   .1521118   .0355767      .0961788    .2405728
            cov(year0,_cons) |   -.010395   .0072502     -.0246051    .0038151
-----------------------------+------------------------------------------------
childid: Unstructured        |
                  var(year0) |   .0114371   .0019599      .0081744    .0160021
                  var(_cons) |   .4920312   .0329821       .431454    .5611137
            cov(year0,_cons) |   .0175375   .0062609      .0052664    .0298087
-----------------------------+------------------------------------------------
               var(Residual) |   .2986372   .0065957      .2859857    .3118484
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 5468.63              Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. lrtest . retained_varies

Likelihood-ratio test                                 LR chi2(6)  =     14.79
(Assumption: . nested in retained_var~s)              Prob > chi2 =    0.0220

Note: The reported degrees of freedom assumes the null hypothesis is not on
      the boundary of the parameter space.  If this is not true, then the
      reported test is conservative.

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |      7,230          .  -8154.673      10    16329.35   16398.21
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.
```

Now we can add some explanatory variables on level 2 and cross-level interactions. I also attempted to allow slopes of level 2 variables to vary, but only BLACK seems to have any sizeable slope variation, so I included it here. Also, covariances among random effects of slopes on school level had to be constrained to be 0 (they appeared very close to 0, and wouldn't estimate otherwise in this model with black and child-level covariances).

```
. mixed math c.year0##i.female c.year0##i.black c.year0##i.hispanic i.retained
##i.female i.retained##i.black i.retained##i.hispanic || schoolid: year0
retained black || childid: year0 retained, cov(unstr)
```

```
Mixed-effects ML regression                     Number of obs     =      7,230
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
       schoolid |         60         18      120.5        387
        childid |      1,721          2        4.2          6
-------------------------------------------------------------

                                                Wald chi2(11)     =    3245.78
Log likelihood = -8116.7578                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |   .7770095   .0211966    36.66   0.000     .7354649    .8185541
    1.female |  -.0515563   .0473707    -1.09   0.276    -.1444012    .0412886
             |
     female#|
    c.year0 |
          1  |   .0150246    .011759     1.28   0.201    -.0080226    .0380718
             |
       year0 |          0  (omitted)
     1.black |  -.4544733   .0883343    -5.14   0.000    -.6276054   -.2813412
             |
      black#|
    c.year0 |
          1  |  -.0492823    .021027    -2.34   0.019    -.0904944   -.0080702
             |
       year0 |          0  (omitted)
  1.hispanic |  -.4798107   .0969086    -4.95   0.000     -.669748   -.2898734
             |
   hispanic#|
    c.year0 |
          1  |    .036259   .0241381     1.50   0.133    -.0110509    .0835688
             |
  1.retained |  -.0514953   .1233298    -0.42   0.676    -.2932173    .1902267
             |
   retained#|
     female |
        1 1  |   .0909304   .0677083     1.34   0.179    -.0417755    .2236362
             |
   retained#|
      black |
        1 1  |   .1106674   .1286472     0.86   0.390    -.1414765    .3628113
             |
   retained#|
   hispanic |
        1 1  |   .0534891   .1533108     0.35   0.727    -.2469946    .3539729
             |
       _cons |  -2.307613   .0813587   -28.36   0.000    -2.467073   -2.148153
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolid: Independent        |
                  var(year0) |   .0081062   .0019893       .005011    .0131131
               var(retained) |    .026531   .0158145      .0082484    .0853363
                  var(black) |   .0487207   .0463472      .0075506     .314374
                  var(_cons) |   .0877577   .0349077      .0402438    .1913687
-----------------------------+------------------------------------------------
childid: Unstructured        |
                  var(year0) |   .0119105   .0020023      .0085671    .0165587
```

```
             var(retained) |   .0237611   .0366761      .0011535    .4894708
                var(_cons) |   .4877012   .0332189      .4267521    .5573551
      cov(year0,retained) |   .0029577   .0072705     -.0112922    .0172076
         cov(year0,_cons) |   .0148103   .0063953      .0022757    .0273448
      cov(retained,_cons) |   -.087717   .0390159     -.1641867   -.0112473
-----------------------------+------------------------------------------------
             var(Residual) |    .296089   .0067807      .2830929    .3096817
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 5000.43            Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Next, we can consider exploring level 3 predictors. Let's mean-center and aggregate:

```
. egen tag=tag(schoolid)

. for var size lowinc mobility: sum X if tag==1 \ gen Xm=X-r(mean)

->  sum size if tag==1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        size |        60    642.5333    317.3676        113       1486

->  gen sizem=size-r(mean)

->  sum lowinc if tag==1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      lowinc |        60    73.73667    27.27405          0        100

->  gen lowincm=lowinc-r(mean)

->  sum mobility if tag==1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    mobility |        60      34.745    13.20786        8.8         67

->  gen mobilitym=mobility-r(mean)

. for var retained female black hispanic: bysort schoolid: egen X_sch=mean(X)

->  bysort schoolid: egen retained_sch=mean(retained)

->  bysort schoolid: egen female_sch=mean(female)

->  bysort schoolid: egen black_sch=mean(black)

->  bysort schoolid: egen hispanic_sch=mean(hispanic)

. for var retained_sch- hispanic_sch: sum X if tag==1 \ gen Xm=X-r(mean)

->  sum retained_sch if tag==1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
retained_sch |        60    .0554006    .0670196          0   .3714286

->  gen retained_schm=retained_sch-r(mean)
```

```
->  sum female_sch if tag==1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
  female_sch |         60    .4984854    .1116362   .1428571    .744186

->  gen female_schm=female_sch-r(mean)

->  sum black_sch if tag==1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
   black_sch |         60    .5734986    .4109914          0          1

->  gen black_schm=black_sch-r(mean)

->  sum hispanic_sch if tag==1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
hispanic_sch |         60    .2031471    .2850104          0          1

->  gen hispanic_schm=hispanic_sch-r(mean)

. mixed math c.year0##i.female c.year0##i.black c.year0##i.hispanic
i.retained##i.female i.retained##i.black i.retained##i.hispanic sizem lowincm
mobilitym retained_schm female_schm black_schm hispanic_schm || schoolid: year0
retained black || childid: year0 retained, cov(unstr)

Mixed-effects ML regression                     Number of obs     =      7,230
-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
       schoolid |         60         18      120.5        387
        childid |      1,721          2        4.2          6
-------------------------------------------------------------

                                                Wald chi2(18)     =    3435.06
Log likelihood = -8100.6828                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year0 |   .7789677   .0211264    36.87   0.000     .7375608    .8203746
    1.female |  -.0588736   .0475313    -1.24   0.215    -.1520333     .034286
             |
     female#|
     c.year0 |
          1  |   .0157074   .0117746     1.33   0.182    -.0073703    .0387852
             |
       year0 |          0  (omitted)
     1.black |  -.4507678   .0950697    -4.74   0.000     -.637101   -.2644345
             |
      black#|
     c.year0 |
          1  |  -.0514084    .021239    -2.42   0.016    -.0930361   -.0097806
             |
       year0 |          0  (omitted)
  1.hispanic |  -.4231151   .1020328    -4.15   0.000    -.6230957   -.2231345
             |
   hispanic#|
     c.year0 |
          1  |   .0316889   .0242593     1.31   0.191    -.0158585    .0792363
```

```
             |
  1.retained |  -.0393028   .1244782    -0.32   0.752    -.2832756      .20467
             |
    retained#|
      female |
         1 1  |    .089565   .0677085     1.32   0.186    -.0431413    .2222713
             |
    retained#|
       black |
         1 1  |   .1145005   .1299947     0.88   0.378    -.1402845    .3692855
             |
    retained#|
    hispanic |
         1 1  |   .0571244   .1542868     0.37   0.711    -.2452722     .359521
             |
       sizem |  -.0001194   .0001421    -0.84   0.401    -.0003979    .0001591
     lowincm |  -.0045189   .0026423    -1.71   0.087    -.0096977    .0006598
   mobilitym |  -.0046824   .0036244    -1.29   0.196    -.0117861    .0024214
retained_s~m |  -3.333575   .7736528    -4.31   0.000    -4.849907   -1.817244
 female_schm |   .1893789   .4174829     0.45   0.650    -.6288726     1.00763
  black_schm |    .490364   .2572757     1.91   0.057    -.0138871     .994615
hispanic_s~m |   .3489856    .341413     1.02   0.307    -.3201715    1.018143
       _cons |  -2.318878   .0820417   -28.26   0.000    -2.479677   -2.158079
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolid: Independent        |
                  var(year0) |   .0076979   .0019595      .0046741    .0126778
               var(retained) |   .0319476   .0172229       .011106    .0919006
                  var(black) |   5.61e-13   5.79e-10             0           .
                  var(_cons) |   .0565938   .0162535      .0322335    .0993644
-----------------------------+------------------------------------------------
childid: Unstructured        |
                  var(year0) |   .0120101   .0020084      .0086537    .0166682
               var(retained) |   .0177631   .0357394      .0003443    .9165099
                  var(_cons) |   .4877259   .0331133      .4269578     .557143
         cov(year0,retained) |   .0031477   .0072575     -.0110766    .0173721
            cov(year0,_cons) |   .0147692   .0064009      .0022238    .0273147
         cov(retained,_cons) |  -.0765241   .0388023     -.1525753    -.000473
-----------------------------+------------------------------------------------
               var(Residual) |   .2959666    .006776      .2829794    .3095499
------------------------------------------------------------------------------
LR test vs. linear model: chi2(10) = 4499.42          Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

We could further examine cross-level interactions involving these three level predictors (we can do two-way interactions with level 1 and level 2 variables as well as three-way interactions across all levels). We can also see that variance for black slope should probably be fixed after all.